Contributions to Economics

Thorsten Wichmann

Agricultural Technical Progress and the Development of a Dual Economy

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To Silke

Preface

This study was written as my Ph.D. thesis at the Technische Universität Berlin. Many people have contributed to this work in one way or another. I am deeply indebted to all of them.

The thesis would still be unfinished, had not Gernot Weißhuhn given me the time and freedom to concentrate on its completion. My thesis advisers, Jürgen Kromphardt and Georg Meran, helped with numerous valuable suggestions, remarks and warm encouragements. Laszlo Goerke accompanied the development of this study over the years with helpful comments on various versions. It has also benefitted from joint work with him and from Manfred Holler's comments on this work. Pio Baake helped to get the modelling part right. At one stage or another also Gesa Bruno-Latocha, Maria Kraft, Beate Scheidt, and Jessica deWolff read and commented on parts of the thesis. Last but not least the University of Siena workshop on "Endogenous Growth and Development" and conversations in the very special atmosphere provided a motivation to continue the work.

Of course I am the only one to blame for all remaining errors and imperfections.

Berlin, May 1996

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List of Variables Used in Chapters 3-6

A	state of technology in agriculture
K	capital stock
L	labor
M	state of technology in manufacturing/industry
Y_A	level of food production
Y_M	level of widget (industrial/manufacturing good) production
a	fraction of adopted technologies
c_A	per-capita food consumption
c_M	per-capita widget consumption
h	level of human capital
m	rate of technology adoption
n	fraction of labor in agriculture
u	fraction of agricultural labor in production
z_1	control-like variable
z_2	state-like variable
Π	level of nutrition caused productivity
α	output elasticity of labor
δ	productivity parameter in human capital accumulation and research
ε	income elasticity of demand
η	output elasticity of agricultural technology
γ	weight of food consumption in utility function
λ	labor force growth rate
ν	growth rate of A
μ	growth rate of M
ϕ	productivity parameter in human capital accumulation and research
π	upper limit for Π
ρ	discount factor for utility
σ	inverse of intertemporal elasticity of substitution
θ	shadow price in optimal control problems
ξ	subsistence consumption

Unless stated otherwise, subscripts A and M denote agriculture and manufacturing/industry, respectively.

> It is in the [agricultural] sector that the battle for long-term economic development ... will be won or lost.
>
> *Gunnar Myrdal*[1]

1. Introduction

Inquiries into the "Nature and Causes of the Wealth of Nations" (*Smith*, 1776) are as old as the history of economic thought. Periodically, interest revives for the questions why some countries are poor and others rich, some industrialize and others remain agricultural, and why some countries have high growth rates of GDP while others experience even a decline. The last wave of research, which is by now well known under the heading "New Growth Theory" (NGT), began with work of *Romer* (1986) and *Lucas* (1988). It finished a period of tacit consensus among economists that problems of developing countries are dealt with in the field of development economics while growth theory, if it has any relevance at all, is only applicable to growth behavior of modern, industrialized economies. Since then, economists are again on the search for a single theory explaining growth and development of rich and poor countries alike.

While this single theory has not been found yet, the NGT has improved our understanding of the role that human capital accumulation as well as research and development (R&D) play in the growth process. It has also pointed to the consequences that externalities from these activities have. Since these topics are important for developed and for developing countries alike, a huge amount of empirical and theoretical studies emerged, each illuminating a different facet of growth and development.

However, the way in which many of these studies analyze diverging growth experience seems dubious in many respects. Most of the studies are more or less elaborate extensions of neoclassical growth theory. They neglect the theoretical as well as the empirical knowledge about the development process which has been accumulated over the past decades, mainly in development economics. The most serious of these omissions – this is the central claim behind this study – is ignoring

1. *Myrdal* (1968, 1241).

the fundamentally different nature of developing economies. Within these economies agriculture has a significance it has long lost in industrialized countries: in many countries more than two thirds of the population work in agriculture, food consumption is often close to, in some cases even below subsistence, and availability of arable land represents a serious constraint. The industrialized sector, on the other hand, is small compared to agriculture and less dominant than in industrialized countries. Contrary to these findings much of the NGT assumes that developing countries are not fundamentally different from industrialized countries, just smaller in some sense: they have less physical and human capital, fewer innovations, or a less efficient tax system.

In this study we choose the opposite approach. Building on work done in the field of development economics, especially on the dual economy model, we derive a two-sector model of a developing country to study growth and structural change within this kind of economy. The derived economy is dual in the sense that it consists of two asymmetric sectors, traditional agriculture and modern industry. The first asymmetry is in production: in agriculture the output is produced with labor and land, while the industrial input factors are labor and capital. The second asymmetry is in consumption. Food is necessary for staying alive which is not generally the case with industrial goods. Nutrition can also influence productivity. Therefore the output of agriculture, food, plays a special role in the development process. Compared to one-sector models, this two-sector approach not only has the advantage of being more realistic; it allows the analysis of structural change as well.

While based on the dual economy hypothesis, the analysis conducted here is also part of the NGT since we discuss *questions* emphasized in this strand of growth theory with *tools* which have been developed in this literature. Within the dual economy framework we study three different topics, all located in the agricultural sector. Contrary to most other studies, which neglect agriculture, we focus on this sector. The first topic is technical progress in agriculture. We present a model where the rate of technical progress is endogenized as in familiar NGT models. Within this context questions of schooling and agricultural research in developing countries are discussed. The second topic is technology adoption in agriculture and the catch-up process to technological leaders. While these two topics concentrate on the production in agriculture, hence the first asymmetry, the third discusses consequences from the specific character of the agricultural output food, the second asymmetry.

Compared to the rich literature on growth and development we only consider a rather limited set of questions: The focus here is on growth (defined as increases in per-capita consumption of the two goods produced in the economy) and on structural change (defined as changes in the fractions of labor in each sector) in a closed economy. We denote the simultaneous occurrence of non-negative long-run growth in both sectors together with a structural change increasing the fraction of labor in industry as development of a country. Thus, all other equally important elements of development like, for example, health, democratic institutions, or integration into the world economy cannot be considered here.

Also the regional focus is limited. The stylized economy we have in mind is a *very* underdeveloped country where agricultural productivity is very low and can be increased by better technologies. This is the case in several African and Asian countries. The analysis thus excludes most South-American developing economies, where distributional considerations are more important than technological, and it excludes industrializing countries like Taiwan or South Korea which are beyond the stage discussed at this place.

Since many of the questions discussed here are only relevant at low levels of income or only for a certain time, a two-step approach of algebraic analysis and numerical simulation is taken: In the first step the long-run steady-state is calculated algebraically and its determinants are discussed. In the second step the transitional dynamics towards this steady-state, which might last for a rather long time, are calculated with numerical simulations. For this purpose we apply a new method which has also been developed within the NGT and extend it to the problems discussed here.

The combination of algebraic and numerical solutions has so far only rarely been chosen in economic analysis. It does have, however, some advantages: Compared to a purely algebraic discussion on the one hand, it takes into account the possibly lengthy transition period towards a new steady-state which might be more relevant for economic policy than the distant steady-state itself. In addition numerical

numerical calculations allow us to compare the model's outcome with stylized facts of development: facts about quantitative characteristics of major variables during the development process as well as about the relevant time-scale. Compared to a purely numerical analysis, on the other hand, the two-step approach ensures at least some knowledge about the dynamics within the model, whereas the former is often a black box.

The analysis is conducted within five chapters. Chapter 2 contains an introduction into the literature. In its first part the main characteristics of the NGT are pointed out to position the study conducted here within the literature. It is described how endogenous growth mechanisms work and how these mechanisms can be used to explain characteristics of economic development. The second part presents insights from the development economics literature about the role agriculture plays in the process of economic development. Both parts together provide the motivation of this study.

In chapter 3 a simple two-sector model of agriculture and industry in a dual economy is presented which builds on *Jorgenson* (1961). The model is set up as an optimal control problem with exogenous technical progress in both sectors. It is a baseline model for the subsequent chapters. Its properties, the steady-state solution as well as stability and uniqueness of the equilibrium, are investigated. Last, some economic policy experiments are conducted which seem reasonable in the light of the insights from development economics presented in chapter 2. These outcomes are compared to stylized facts of economic development.

Chapter 4 contains an extension of the baseline model to endogenous technical progress in agriculture along the lines of *Lucas* (1988). This extension allows a more detailed investigation of determinants of productivity increases and therefore a better combination with insights about productivity improvements collected in the development economics literature. The focus is here on technology creation on the one hand and on human capital accumulation on the other hand. Both have been extensively studied in development economics. With these extensions the model's steady-state and stability of equilibrium are analyzed and compared to the baseline model. As in the previous chapter, some policy experiments are conducted and the results compared to the stylized facts.

While chapter 4 discusses the *creation* of technologies, chapter 5 is concerned with technology *adoption* from technological leaders and with the catch-up process. A very popular hypothesis in the empirical analysis of growth and development is convergence: It asserts that underdeveloped countries could make use of freely available technologies of advanced economies and catch up to their consumption levels and growth rates by adopting these technologies. We modify the models from chapter 3 and chapter 4 for a discussion of exogenous and endogenous technology adoption in agriculture. The results are confronted with the convergence hypothesis.

Chapter 6 extends the baseline model in another direction. While the previous chapters have discussed special characteristics of agricultural *production*, this chapter shows special characteristics of food *consumption*. The first effect incorporated into the model is a less than unitary income elasticity of food demand – Engel's law – via subsistence consumption. The second effect is a positive relationship between the level of food consumption and labor productivity. Both effects are empirically well-supported. We analyze how the model's dynamics change if these effects are taken into account. Special attention is given to the question whether their existence together with a technologically stagnant agricultural sector can explain a permanently low degree of industrialization.

Chapter 7 concludes and summarizes the analysis.

2. Economic Development, Endogenous Growth, and Agriculture

This chapter reviews the literature upon which the subsequent analysis is based. We start in the first section by describing the different mechanisms of endogenous growth and then show in the second section where this new methodology is applied to questions of economic development. As a counterpoint to this new analysis of development, we present in the third section some older discussions about the role of agriculture in economic development which is largely neglected in the new work. These discussions originate in development economics rather than in growth theory. In the last section we show how the new ideas can be combined with the older insights and set the agenda for the subsequent chapters.

2.1 Mechanisms of Endogenous Growth

Most endogenous growth models can be put in one of two classes: either they are in a neoclassical fashion and growth is driven by accumulation of capital, human capital, or knowledge. Then, together with *Marshall*ian (1920)[1] externalities, this accumulation generates perpetual growth. Or the models are neo-*Schumpeter*ian (1926), where growth is driven by dynamic entrepreneurs searching for monopoly profits from newly invented products. Since the last years have produced numerous reviews of the so-called "New Growth Theory" (NGT) or "endogenous growth theory"[2] and already two textbooks (*Grossman* and *Helpman*, 1991a; *Barro* and *Sala-i-Martin*, 1995), this overview will only give a short idea of the main model properties.

2.1.1 Accumulation Driven Growth

A large part of the early work on endogenous growth is a simple extension of the orthodox-neoclassical model as pioneered by *Solow* (1956, 1957).[3] Its heart is a linear-homogeneous production function $Y = F(K, L)$ with positive but diminishing returns to both input factors labor (L) and capital (K). A constant fraction s of

1. Here: Book IV, Chapter 9, Section 7.
2. Among others are: *Brander* (1992), *Buiter* (1991), *Fischer* (1993), *King*, *Plosser* and *Rebelo* (1988), *Helpman* (1992), *Lessat* (1994), *Shaw* (1992), *Stern* (1991), *Verspagen* (1992).

output Y is being saved. All savings are reinvested and therefore increase the capital stock:[4]

$$(2.1)\quad \dot{K} = \frac{\partial K}{\partial t} = sF(K, L)$$

The growth rate of output per-capita ($y = Y/L$) is determined by the marginal product of capital. For a constant labor force the growth rate is

$$(2.2)\quad \frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} = sF_K(K, L)\,.$$

Due to the diminishing returns to capital this growth rate goes to zero over time. However, this is not what one observes in reality. *Kaldor's* (1961) first stylized fact about the "advanced capitalist countries" is the existence of steady growth of output per worker. To replicate this behavior, the marginal product of capital in the model must be prevented from falling to zero: In the long run there must be $F_K(K, L) > 0$. An equivalent result can be obtained for a growing population. Here the lower bound on the marginal productivity of capital must be even higher. For a Cobb-Douglas production function, where α is defined as output elasticity of capital and λ as the growth rate of labor, this condition becomes $F_K(K, L) > \alpha\lambda / s$.[5] Therefore the conclusion is that per-capita output only grows without bound if in the process of capital accumulation its marginal product does not decrease too much.

In the orthodox-neoclassical model this is assured by introducing exogenous technical progress. If technical progress is labor-augmenting, the production function changes into $Y = F(K, AL)$ where A denotes the state of technology. Exogenous

---

3. To distinguish this kind of exogenous neoclassical growth model from the new, endogenous neoclassical models, we call the former "orthodox-neoclassical".
   Solow's model has later been refined by *Cass* (1965) and *Koopmans* (1965) in an optimal control framework which builds the foundation of most of the newer analysis. This extension makes the savings rate endogenous and derives saving and consumption from utility maximization.
4. A dot always denotes a time derivative.
5. $\frac{\dot{Y}}{Y} = \frac{F_K \dot{K}}{Y} + \frac{F_L \dot{L}}{Y}$

   $\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = sF_K + \lambda\left(\frac{F_L L}{Y} - 1\right) = F_K\left(s - \lambda\frac{K}{Y}\right)$ by the Euler exhaustion theorem

   $= F_K\left(s - \lambda\frac{\alpha}{F_K}\right)$ by the assumptions about $F(K, L)$.

technical progress increases labor productivity over time with the growth rate ν according to

(2.3) $A(t) = A_0 e^{\nu t}$.

It is labor augmenting and therefore increases the marginal product of capital, offsetting the capital accumulation effect. Not considering the level of technology, the production function is still linear-homogeneous in K and L. Considering the level of technology as a further factor of production, however, there are increasing returns to scale. If capital and labor are paid their marginal products, there is no output left to compensate technical progress, as *Euler's* exhaustion theorem for linear-homogeneous functions shows. Therefore technical progress has to "fall like manna from heaven". In the orthodox-neoclassical model this happens by modelling it as a constant exogenous time trend as in equation (2.3). Then the growth rate of per-capita income equals the rate of technical progress.

If technical progress is to be endogenized as the result of deliberate actions of individuals, the irreconcilability of per-capita growth in output and compensation of each factor with its marginal product becomes a problem. In most neoclassical models of *endogenous* growth this problem is avoided by introducing *Marshall*ian externalities as a by-product of investment. This externality – which takes over the role of technical progress – does not have to be compensated. Growth of per-capita output is not any more determined exogenously but rather driven by investment decisions.

Endogenizing technical progress can be done in several ways. The different kinds of neoclassical endogenous growth models can best be illustrated with the help of the following Cobb-Douglas type production function:

(2.4) $Y = AK^{\alpha}L^{\beta}\bar{K}^{\gamma}$

where K is the accumulated factor. This is usually capital, but can as well be understood as human capital or a mixture of both. L denotes labor as before. The difference between the orthodox-neoclassical model and the NGT approach is the accumulation externality $\bar{K}$. The former is characterized by $\gamma = 0$ and in addition by the following restrictions on the output elasticities: $\alpha, \beta > 0$ and $\alpha + \beta = 1$.

One of the early "new" growth theorists was *Arrow* (1962) with his model of learning by doing which became popular in the version of *Sheshinsky* (1967), decades before the term "New Growth Theory" was coined. In *Arrow's* model technical knowledge grows with the capital stock. This behavior is based on the idea that workers learn whenever new capital goods are introduced into production. Over time they gain experience in using these new goods and increase their productivity. The larger is capital investment, the larger are the possibilities to learn and the faster does productivity increase. *Arrow* emphasized that learning will only take place through the attempt to solve problems. Since the number of problems encountered usually decreases when a technology has been used for a longer time, sustained learning requires the continuous introduction of new capital goods.

Arrow assumes a production function $Y = F(K, AL)$ which is linear in K and L. Based on empirical evidence about the learning process in the American air craft industry he models productivity as developing according to:

$$A(t) = [K(t)]^{\eta} \quad \text{with} \quad 0 < \eta < 1 \tag{2.5}$$

Equation (2.5) shows the externality of capital accumulation. Going back to the Cobb-Douglas production function presented above, *Arrow's* model can be characterized by the parameters $0 < \alpha < 1$, $\beta = 1 - \alpha$, and $\gamma = \eta\,(1 - \alpha)$ since

$$\begin{aligned} &F(K, AL) = K^{\alpha}(AL)^{1-\alpha} = K^{\varphi}L^{1-\alpha}, \quad 0 < \alpha < 1 \\ &\text{with} \quad \varphi = \alpha + \eta(1-\alpha) < 1, \\ &\text{and} \quad \varphi + (1-\alpha) = 1 + \eta(1-\alpha) > 1. \end{aligned} \tag{2.6}$$

This production function has increasing returns to scale as a whole, but a decreasing marginal product of capital. This assures the existence of a steady-state equilibrium where output and capital grow with the same constant rate.[6] This growth rate is here given as $\dot{Y}/Y = \lambda/(1-\eta)$, therefore output per-capita increases.

In *Arrow's* model a prerequisite for growth is not any more exogenous technical progress but a growing population. Obviously this is not in accordance with stylized facts of growth in industrialized countries; some of these are characterized by

6. It has recently been shown that the dynamics of *Arrow's* model are much richer than this. Cf. *d'Autume* and *Michel* (1993) or *Greiner* and *Hanusch* (1994).

an almost constant population and still high growth rates of per-capita income. *Romer* (1986) regards this as a reason, why for a long time no further attempts had been made to endogenize growth.

With the pioneering work of *Romer* (1986) the interest in endogenous technical progress reawakened. He considers his work as based on *Arrow* and introduces some modifications that allow per-capita income growth also with a stagnant population. First, he lifts the limitation of considering only steady-state growth paths and analyzes all kinds of balanced growth paths: capital and output need not grow with a constant and equal rate any more, only with the same rate. With this modification he can look at the case of an increasing marginal product of capital. In the above equation (2.4) the *Romer* model would be characterized by $\alpha, \beta, \gamma > 0$, $\alpha + \beta = 1$, $\alpha + \gamma > 1$. As a second modification *Romer* emphasizes accumulation of technical knowledge rather than physical capital. The latter does not even exist in his model. K stands for knowledge which can be produced with existing knowledge and output by engaging in research. The variable L in equation (2.4) does include all fixed factors like labor and land.

Under certain conditions a balanced growth path with a rising growth rate over time exists. This is achieved by a simple extension which can be found in almost all NGT models, namely a second production function for the accumulated factor knowledge which differs from those for the consumption good. (Recall that in the orthodox-neoclassical model the consumption good and capital are produced according to the same production function.) An equilibrium then requires strong diminishing returns in the production of knowledge. However, an explicit solution for the growth rate like in *Arrow*'s model is not possible.

Romer, like most other proponents of the NGT, considers two kinds of equilibria: the first is a market equilibrium which exists under perfect competition and private utility maximization. The second is a social planner's optimization problem. Due to externalities – firms do not take into account that their research decision, which increases their knowledge, also increases the stock of knowledge available to the whole population – only the social planning solution is Pareto optimal. In the market solution firms accumulate less knowledge than optimal which could justify appropriate government policies.

Lucas' (1988) model is quite similar to *Romer's*. However, he emphasizes the accumulation of labor augmenting human capital rather than productive knowledge. In his model output is produced according to the following production function:

$$(2.7) \quad Y = AK^{\alpha}(uhL)^{1-\alpha}\bar{h}^{\gamma}, \qquad 0 < \alpha < 1, \gamma \geq 0$$

where K and L are physical capital and labor. The level of human capital is denoted by h and $\bar{h}$ denotes the average level of human capital in the economy which is considered exogenous by each firm or individual. It captures the idea that a larger average value of human capital facilitates communication within production and thus increases productivity. In dynamic equilibrium $\bar{h} = h$. Besides actual production individuals can as well engage in human capital accumulation, that is, go to school. If they spend the fraction u of their working time in actual production and the remainder in school, they accumulate human capital according to

$$(2.8) \quad \dot{h} = h\delta(1-u)$$

where δ is a productivity parameter. Equations (2.7) and (2.8) lead to a steady-state equilibrium with increasing per-capita income, even with a constant population. For the existence of such an equilibrium it does not matter whether there exists an externality, that is, whether $\gamma > 0$.

Note that this model works the opposite way of *Romer's*. In *Lucas'* model there are decreasing returns to each accumulated factor but constant returns in the production function of human capital. Therefore human capital can increase continuously which has a labor augmenting effect and keeps the marginal product of capital from falling to zero.

Finally, the simplest neoclassical NGT model is the so-called *AK*-model which was proposed in the work of *King* and *Rebelo* (1990) and *Rebelo* (1991).[7] In equation (2.4) this model is represented by $\alpha = 1$ and $\beta = \gamma = 0$. Then the production function is linear in the accumulated factor: $Y = F(K) = AK$.

7. *Rebelo* notes that this idea is based on a much older idea presented by *Knight* (1935). Another early endogenous growth model where the marginal product of capital is bounded from below by a constant has been proposed by *Jones* and *Manuelli* (1990).

In this context K is defined in a broad sense. It contains all factors which can be accumulated: capital, knowledge, as well as human capital. The marginal product of capital in this model is constant (A). A steady-state equilibrium exists. For a constant A, output grows with the rate of capital accumulation which in turn depends on the savings rate:

$$(2.9)\quad \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{sF(K)}{K} = sA$$

Technical progress like in the orthodox-neoclassical model does not exist. As discussed above, it is not necessary for unlimited per-capita growth of output since the marginal product of capital is assumed constant in this model and cannot fall to zero.

These four models are the foundation of the new growth and development work in the neoclassical tradition. Contrary to the orthodox-neoclassical model the new approach asserts a connection between thriftiness (i.e., the savings rate) and *growth rates*, while the former allows only *level effects* in the long run.

2.1.2 Innovation Driven Growth

The second strand of the NGT emphasizes innovations rather than human capital accumulation or learning. While neoclassical models assume perfect competition, and productivity is (at least partially) increased by externalities without any deliberate effort, models of innovation driven growth put *Schumpeter's* (1926) dynamic entrepreneur at the center of their analysis. He is searching for profit opportunities from innovations and drives technical progress by innovating. Contrary to neoclassical models, market power is explicitly considered: in selling his ideas on the market the innovator is monopolist. However, there also exist parallels to the neoclassical model: as in the latter, production functions for knowledge and the consumption good differ. Also externalities of knowledge accumulation exist and are even necessary to generate growth.

The first model of this kind has also been presented by *Romer* (1990). It has later been refined and extended, particularly by *Grossman* and *Helpman* (1991a) and by *Aghion* and *Howitt* (1992). In addition to knowledge creation by inventions *Aghion* and *Howitt* especially emphasize *Schumpeter's* idea of creative destruction.

Although there exist differences between the models, the basic ideas behind them are similar. Since these models have been reviewed at a number of places, we only give the main ideas.[8]

The economy is made up of two sectors of production.[9] Within the first sector a consumption good is produced under perfect competition from an increasing set of intermediate goods[10] and labor. In this production the intermediate goods are imperfect substitutes. They are produced in a second sector where dynamic entrepreneurs invent new products by hiring labor. Each entrepreneur is monopolist for his invention in selling it to the consumption good sector.[11] Then the elasticity of substitution of intermediate goods in the production of consumption goods can be interpreted as monopoly power of the inventor who sells these goods.

Innovations are based on new designs which are blueprints for production. This is where externalities come in: designs increase the knowledge available to society, knowledge is cumulative. It is available to everybody – also to new inventors, and it is assumed to increase their research productivity. Entrepreneurs do not take this positive externality into account when making their decision whether to innovate or not. Their calculation is only influenced by the revenue they expect from innovations and the cost to finance them via the capital market. This revenue decreases over time as more intermediate goods are used to produce the consumption good. Each new intermediate good takes away part of the profit stream from older innovations. The size of this decrease depends on the market power an intermediate good monopoly has. Since innovations are financed via the capital market, the interest rate also influences the decision to innovate.

These considerations on part of the entrepreneur can be summarized in an equilibrium condition, the so-called "Schumpeter-line": It requires that the value of a successful innovation will reflect the expected present value of profits, where these

8. The most complete treatment of innovation based growth is *Grossman* and *Helpman* (1991a). A summary is given in *Helpman* (1992).
9. It is possible to add a third, traditional sector to this model. This is done, e.g., in *Fung* and *Ishikawa* (1992). Growth in this economy is only driven by the industrial sector through the introduction of intermediate goods.
10. Sometimes this increasing number of intermediate goods is interpreted as increasing division of labor.
11. Thus, innovation is product innovation. There exist modifications, though, that discuss process innovations instead.

profits are reduced by anticipated subsequent inventions of new techniques. A second equilibrium condition is a resource or labor allocation condition which requires that the sum of labor in research and in consumption good production must equal total labor supply. Both conditions together determine the economy's growth rate. This rate is for example higher, the lower the interest rate (less profitable innovations can be conducted) or the higher the degree of monopolization (the value of an innovation decreases more slowly, making it more profitable).

One of the main issues discussed within this framework are the consequences of trade for the growth performance of countries or, as *Grossman* and *Helpman* (1991a) called their book: "Innovation and Growth in the Global Economy". But the analysis is not confined to this subject. Since the models explicitly include the innovation process and are rather detailed, many questions, which had already been discussed in a static framework in fields like industrial organization or in the patent literature, are now discussed in a growth context.[12] These include innovation and imitation (*Segerstrom*, 1991), intellectual property rights (*Helpman*, 1993), and even growth and unemployment from the introduction of new production techniques (*Aghion* and *Howitt*, 1994). All this led even *Solow* to the statement that this strand of the literature "... has an air of promise and excitement about it." (*Solow*, 1994, 52)

2.2 The New Analysis of Development Problems

Neoclassical growth theory used to be rather modest. Since *Solow* (1956) published the first neoclassical growth model it had always been understood that these models could at most give some insights into the growth process of industrialized capitalist countries like the United States. They were considered pure theory and not applicable to development issues of technologically backward economies. This restriction remained commonly accepted in the economics profession for several decades, as surveys (e.g., *Hahn* and *Matthews*, 1964) and textbooks (e.g., *Hacche*, 1979, 25) show alike.

12. *Feenstra* and *Markusen* (1994) in addition have recently shown that this kind of models can generate the kind of results that are obtained from growth accounting studies.

This consensus has been broken up by the "New Growth Theory" through the seminal work of *Romer* (1986) and *Lucas* (1988), who's Marshall lecture was titled: "On the Mechanics of Economic Development". Lucas was fascinated by problems of economic development:

> Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, *what* exactly? If not, what is it about the 'nature of India' that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else. (*Lucas* 1988, 83)

He therefore demands that theory should be able to reproduce

> the main features of the world economy: very wide diversity in income levels across countries, sustained growth in per-capita incomes at all income levels..., and the absence of any marked tendency for growth rates to differ systematically at different levels of income. (*ibid.*, 118)

In search of "a new paradigm of economic growth" (*Ehrlich*, 1990, S1) several hundred papers have followed since then extending the first models or investigating their properties, analyzing similar questions with different tools, applying the new methodology to a large variety of questions, or empirically testing hypotheses set up by the new theory. The latter has been especially aided by the publication of the Penn World Table (*Summers* and *Heston*, 1991) which contains a set of major macroeconomic variables on a purchasing power basis for almost all countries of the world. This data set is freely available and has been used widely in so-called "cross-country growth regressions" although the outcome is often somewhat dubious (cf. *Levine* and *Renelt*, 1992).

In the course of this new research program many ideas have been rediscovered or restated in a new theoretical framework, so, for example, big push theories, human capital, or learning by doing. Not always did the new growth theorists know that supposedly new ideas are nothing new to economists who are familiar with the many branches of growth theory from the 1960s or with development economics. This can probably seen as one major reason, why the arrival of the NGT was not unanimously celebrated. Especially "old" growth theorists like *Solow* (1990, 1991, 1994) or *Srinivasan* (1994a) have not considered every part of the NGT a genuine novelty in economic science.

Nevertheless, the NGT has brought questions of economic development back to the center of economic theorizing where they were when Adam *Smith* (1776) published his "Wealth of Nations". This is certainly an improvement of the reputation of development economics which so far had often been considered an inferior field of economics as *Leijonhufvud* (1973) has told us with a wink in his ethnographic study of the Econ tribe. Whether this new research program has led to new insights about problems of economic development, is still debatable and probably far too early to tell. While the proponents of this new direction would certainly give an affirmative answer, development economists are more cautious (cf. *Bardhan*, 1994; *Raut* and *Srinivasan*, 1993; *Srinivasan*, 1994a).

The central ideas embodied in the NGT models presented in the last section – human capital accumulation, learning by doing, innovations, increasing returns, and spillovers – have been used in numerous studies to discuss many different facets of growth and development. Since they sometimes use different methodologies to study similar questions, at other times one methodology to study different questions, and since there are many overlappings between the studies, there is no obvious coherent framework to present them in. There exist some differences between them, though, which appear useful from the point of view of this study. First of all, there is some work which focuses on different *growth performances of countries* and asks what economic, political, or social factors might be responsible for the differences. Secondly, there exist models which primarily emphasize the *structural development* of an economy. These studies discuss the development of an economy from primarily traditional to more advanced methods of production and also the relationship between structural development and growth. And last, there are some studies discussing the interdependence between an *economy's structure* and its *integration into the world economy.*

2.2.1 Growth Performance

One of the factors influencing growth performance has already been pointed out in the basic NGT models, namely thriftiness. Contrary to the orthodox-neoclassical model, the savings rate influences growth positively, not only in the transition period but also in the steady-state. This is due to forces which prevent the marginal productivity of capital from falling to zero. Thus, government policies which induce a higher savings rate could increase growth. This relationship is not limited to the activity traditionally understood as saving in neoclassical models, namely accumulation of physical capital. It rather encloses all kinds of relinquishment of present consumption in favor of future consumption. This also includes going to school to accumulate human capital.

The savings rate mechanism has been explored in several studies on the influence of government expenditure on growth: *Barro* (1990) and *Rebelo* (1991) argue that the empirically observed heterogeneity in cross-country growth experiences could be due to differences in government policy. Within an *AK*-model they show that changes in policy variables like an increase in the income tax rate decrease the rate of return on investment and, since saving equals investment in these models, also the rate of return on saving. Thereby taxing leads to a permanent decline in the rate of capital accumulation and thus the rate of growth. *Lee* (1992) presents an extension of *Barro's* model where the government uses its revenues for income transfers, public investment, and public consumption goods. The latter expenditure enters the individuals' utility functions. If government spending is financed by income taxes, two equilibria exist: one with a high income tax rate, where taxes are mostly used for income transfers and growth rates are low, and a second one with a low income tax rate, where revenues are mainly used for public investment and the growth rate is high.

Saving behavior also plays a major role in the rapidly growing literature on income distribution and growth which often has a public choice perspective. *Bertola* (1993) points out that with heterogeneous agents, who differ by their income share resulting from accumulated factors like physical or human capital, growth increasing policies will have distributional consequences since they work by increasing accumulation. Therefore the implementation of such policies will face political constraints. Growth maximizing policies are only optimal for governments that care solely about "pure capitalists".

If governments are elected in a democratic process, income inequality might be harmful for growth as *Persson* and *Tabellini* (1994) as well as *Alesina* and *Rodrick* (1994) among others have shown: in societies where inequalities are large and hence distributional conflict is more important, elections will often produce a government who's decisions allow less than full private appropriation of returns from investment. Instead they contain redistributional policies, for example, via capital income taxes. These policies decrease the net return on investment and therefore lead to less accumulation and growth.[13] *Persson* and *Tabellini* point out that empirical evidence from cross-country regressions supports the assertion that income inequality is harmful to growth.

Besides saving differences also other things might be responsible for the large diversity in growth rates among countries. *Azariadis* and *Drazen* (1990) point out that there might exist threshold externalities in economic development which lead to multiple equilibria with high and low growth rates respectively. Under certain conditions an economy can get locked into an underdevelopment trap with permanently low growth rates. These externalities may arise through the process of human capital accumulation if reaching a given level of knowledge either makes it easier to acquire further knowledge or induces a sharp increase in production possibilities. A similar intuition is behind the model of *Zilibotti* (1993). *Azariadis* and *Drazen* cite some empirical work which quantifies a threshold connection between human capital and growth rates as suggesting that a literacy rate of at least 30-40 percent might be a precondition for growth.

There are some studies which further investigate the process of human capital accumulation, especially schooling, and possible influences on the growth rate: *Becker, Murphy,* and *Tamura* (1990) analyze societies where families decide between either investing in human capital or establishing large families. *Glomm* and *Ravikumar* (1992) ask the question when an economy prefers private and when public education. They show that individuals vote for public education if a majority of agents have incomes below average. *Pecorino* (1992) investigates the question how agents behave if an economy has rents to distribute. In this model agents have to decide whether to specialize in rent seeking (e.g., by becoming full-time lobbyists) or to pursue rent seeking in addition to their productive activity.

13. These results do not rule out the possibility that without the political process inequality might be favorable to growth. *Galor* and *Tsiddon* (1994), e.g., show that for a poor economy it might be advantageous to subsidize education of a selected group of individuals who will eventually generate large enough externalities from their human capital to increase everybody's income in the long-run.

Rent seeking reduces growth rates if no agents specialize in this activity since it reduces the incentive to accumulate productive human capital.

Studies like those presented here make up the largest part of the New Growth Theory, probably because the underlying simple endogenous growth model is relatively easy to handle, can be extended in many directions, and is close to the orthodox-neoclassical growth model familiar to most economists. However, these studies neglect an important feature of economic development, namely structural change.

2.2.2 Structural Change and the Development of Industry

While the research presented so far considers development primarily as a question of per-capita income growth rates, there also exists some work that emphasizes structural change – in whatever sense – as main characteristic of economic development. All these studies have in common that they focus on the transition from traditional methods of production to modern production techniques of industrialized economies. In contrast to traditional production, these modern methods are characterized by human and physical capital intensiveness as well as knowledge intensiveness, by increasing returns, and by highly specialized firms and labor. Most of the studies focus on one of these characteristics and look at obstacles to the adoption of advanced methods of production.

Within the NGT, *Murphy, Shleifer* and *Vishny* (1989a) as well as *Matsuyama* (1992a) have reconsidered a problem that had first been pointed out by *Rosenstein-Rodan* (1943), namely coordination failures: When domestic markets are small and world trade is not free and costless, firms may not be able to generate enough sales to make the adoption of increasing returns technologies profitable. If, for example, a single shoe factory adopts more productive technologies to expand its output, it would generally not be able to sell all shoes since its workers would not want to spend all their additional income on shoes. There is not enough demand for all the additional output. Therefore the factory would not adopt the new technology unless other sectors of the economy go along with her and adopt more productive technologies, too: such a situation characterizes a coordination failure. The government should coordinate expansion so that the different sectors develop equally, growth is balanced, and the necessary demand is generated.[14]

The coordination failure literature is supported by a strand of the literature that concentrates on the division of labor. First of all, division of labor can be a major source of growth, as already manifested in Adam *Smith's* pin factory and forcefully restated by *Young* (1928). Following this argument *Kim* and *Mohtadi* (1992) construct a model of endogenous growth where growth is only driven by the division of labor. Naturally there exist limits to this division of labor, and in underdeveloped countries these limits might be especially tight. *Becker* and *Murphy* (1992), for example, point out that the division of labor is limited by costs of coordinating the divided activities. These costs might be especially large in poor countries if coordination requires a sophisticated infrastructure. *Yang* and *Borland* (1991) bring several of these issues together and show how interactions among the effects of accumulated experience and specialization of productivity, the effects of transaction costs, and preferences for current and diverse consumption can generate continuous economic growth based on evolution of the division of labor.

Finally, there exists another reason why countries might not be able to adopt sophisticated technologies, namely incomplete or even missing financial markets. Production is risky, and therefore a demand shift might make a whole production facility together with its human capital worthless if both are highly specialized. As *Saint-Paul* (1992) notes, there are generally two ways to insure against this risk: either risk sharing via financial markets or using flexible but less productive technologies which in the worst case can be converted to produce different goods. Therefore he concludes that financial markets and highly productive technologies are strategic complementarities. *King* and *Levine* (1993) support this view with empirical evidence: They show that various measures of the level of financial

14. In a different paper *Murphy, Shleifer* and *Vishny* (1989b) show that the necessary demand can come from a sector which has profited from special events like an export boom for its output. They point out that income distribution in this sector is crucial for the question if the profits lead to the necessary demand increase. If they go to a small upper-class with preferences for hand-crafted or imported goods, domestic demand would only rise by very little. If, in comparison, the profits go to the broad masses with demand for common-day goods which are being produced domestically, demand rises. Therefore an equal distribution of gains from such a historic event is a prerequisite for subsequent development.

development are positively correlated to real per-capita growth, the rate of physical capital accumulation, and improvements in the efficiency of capital usage.

While studies in the fashion described here analyze the development process in much more detail than those presented in the previous subsection, they are still subject to at least two criticisms: first of all, they seek the solution to underdevelopment in rapid industrialization and regard the traditional sector mostly as backward and hopeless. It is shown in section 2.3 that development economists are divided on this assessment. Secondly these models neglect the integration of a developing country into the world economy.

2.2.3 Growth in the Open Economy

One of the most passionately discussed issues in economic development is the question whether opening a country to trade is advantageous. Within the context of growth theory this discussion centers on the dynamic consequences of trade. Are there dynamic gains or losses? Who will win or lose? Does a country which is integrated in the world economy show faster or slower growth? Within the NGT these questions are discussed in two different frameworks, namely in models of innovation driven growth and in models with learning by doing. The typical framework is a two-good, two-country model where one good is produced in a simple traditional way and the other with a modern technique. The latter either requires innovations to increase its productivity or its production to be characterized by the presence of learning externalities.

Within the innovation literature one positive growth effect mentioned is that integrating economies gives incentives to entrepreneurs in each of the countries to invent products that are unique on the world market (*Rivera-Batiz* and *Romer*, 1991). Since the increased market for an innovation raises expected profitability of research, it spurs innovation. If in autarchy knowledge is not freely available, opening a country also makes a larger knowledge base available to researchers. This argument is even more valid if knowledge is to a large extend disseminated by trade (*Grossman* and *Helpman*, 1991b).

However, there are also effects at work which might decrease an economy's growth rate after it has opened to trade. Usually these effects are closely connected to its dynamic comparative advantage. If one country has a comparative advantage to innovate, for example due to a larger amount of human capital or a larger and more advanced research sector, it will specialize in innovating new goods and under certain circumstances also specialize in producing the sophisticated good. The other country, which might be poor in human capital but rich in unskilled labor or natural resources, will specialize in production of the traditional good. As consequence innovation in this country will slow down while the rich country experiences even higher innovation rates (*Grossman* and *Helpman*, 1991a, ch. 9; *Feenstra*, 1990). There is some evidence in favor of this position. As *Grossman* and *Helpman* (1994, 40) note, resource rich countries like Canada or Australia devote far smaller shares of their national output to R&D than resource poor countries at a similar stage of development. However, while the producer of the traditional good has a lower innovation rate, the welfare consequences of integration into the world economy are less clear: consumers in this country might still gain from trade due to the usual static gains and due to a larger variety of goods. The specialization might cause a long-run world welfare loss, though, due to the decreasing research efforts in the poor country.

A similar argument like that from innovation and trade models is presented from the learning by doing literature. Again two sectors are assumed, one traditional and the other advanced. In the advanced sector learning occurs and increases productivity as in *Arrow*'s model. As countries open up for trade, they are locked into the comparative advantage pattern that prevails at that moment (*Boldrin* and *Scheinkmann*, 1988; *Matsuyama*, 1992b). That country which initially has a slight comparative advantage in producing the advanced good intensifies its advantage through productivity increases from learning. *Stokey* (1991) proposed a similar model, where one country produces a high-quality spectrum of goods and the other a low-quality spectrum. Human capital in both countries is acquired by learning. If producing the high-quality goods leads to larger learning effects, opening the countries for trade again raises the growth rate for the high-quality producer and reduces it for the other. *Bardhan* (1994) points out that according to these results subsidizing infant export industries, which produce high quality goods and where large learning effects can take place, might be more growth promoting than a policy of protecting import substitution industries. In the former case the opportunities for learning from newer and more sophisticated goods are larger than in a situation of restriction to the home market.

Young (1991, 1993) has addressed the point that modelling an economy as experiencing continuous learning from production is far too simple. In reality learning is bounded. Whilst in the beginning of producing a new good productivity increases from learning are high, learning peters out when agents become familiar with the new technique. Hence, to keep learning going, an economy has to change the basket of goods it produces continuously.[15] For a country, which is integrated in the world economy, it might therefore make sense to conduct a policy of a "narrow moving band" (*Krugman*, 1987): Closing the market for a good in which foreign countries have a comparative advantage but the home country could experience learning effects allows a country to profit from these effects. Over time this market closure has to move from goods where learning effects have been exhausted to sectors where new learning takes place. *Lucas* (1993) sees this policy as one of the reasons for the economic success of lately developed countries like the Philippines and Korea.

The studies presented cover a broad range of questions crucial to economic development and emphasize several important points. However, almost all of them have one thing in common: they see the driving force of economic development in the industrial or modern sector of an economy. It is therefore only logical that they focus on special characteristics of modern production inhibiting its adoption or on economic policies harming this sector. The NGT helps in identifying some of these characteristics, for example, increasing returns due to externalities, learning effects from the introduction of new techniques, or specific human capital requirements. The analysis in NGT fashion presented includes these special characteristics and shows that their inclusion can indeed lead to underdevelopment traps or multiple, high and low growth, equilibria. Underdevelopment traps or multiple equilibria might be reasons why different countries are at different levels of development and why underdeveloped countries do not catch up to industrial-

15. *Stokey* (1988) also emphasizes the changing set of goods in production and investigates the consequences of a second, traditional sector (agriculture) where no learning occurs. A country without experience in the learning sector remains stagnant forever and specializes in agricultural production. If, however, this economy somehow acquires enough experience in manufacturing, it starts to grow.

ized countries as fast as simple neoclassical growth models seem to imply. The NGT also discusses in depth policies unfavorable to the dynamic sector by inhibiting accumulation of physical or human capital. Taxation of gains from accumulation, be it by accident or caused by pressure from underprivileged voting groups, can be responsible for these policies.

Within this rich literature, though, almost no study can be found where the traditional sector, agriculture, is *not* condemned to remain traditional and stagnant forever.[16] This is hardly in accordance with stylized facts from economic development and difficult to reconcile with policy recommendations given by institutions like the *World Bank*, as the next section shows.

2.3 Some Older Thoughts about Agriculture in Economic Development

One of the main issues of development economics is the structural transformation of a traditional into a modern economy. Its emphasis is also one of the main differences between neoclassical growth theory and the theory of development. While the former concentrates on growth of per-capita income in industrialized countries, the latter has as its subject "the *structure* and behavior of economies where output per head is less than 1980 US $2,000." (*Lewis* 1984, 1, italics added) Although "structural transformation" is a widely used term, it is not easy to define, as *Chenery* (1988, 50) notes. "In general it connotes the set of changes in economic and institutional structures necessary to continued growth of GNP... [It] would include the accumulation of physical and human capital and shifts in the composition of demand, production, trade, and employment."

Beginning with *Kuznets* these phenomena have been studied as a whole set of interdependent questions.[17] He analyzed the development process of several now advanced countries beginning as early as the late eighteenth century and ranging up to the 1950s. His most intriguing result is that agriculture shrinks as income

16. An exception is some work on population and productivity arguing that an increasing pressure of population on scarce land leads to invention of new, more productive technologies. Cf. *Boserup* (1981) as well as *Kremer* (1993). A relationship between population size and factor productivity also exists in *Raut* and *Srinivasan* (1994).

17. His research program started with a series of articles (1956-67) and lead to a summary in *Kuznets* (1966).

rises: For all countries and all time periods in his data set, the agricultural share in the total labor force declined. His second, although less clear observation was a tendency of the share of agricultural output in total production to decline. However, this decline was slower than the labor share decline, so that labor productivity in agriculture rose more rapidly than in the rest of the economy. Comparing average labor productivity in agriculture and industry, *Kuznets* (1966) observed an initial decline in relative labor productivity in agriculture and a subsequent increase, altogether a U-shaped pattern.

While these patterns are mainly from time-series analyses of advanced countries, partly similar results have been obtained from cross-country studies of developing countries (*Chenery* and *Syrquin*, 1975; *Syrquin* and *Chenery*, 1986). These studies find an almost universal inverse association of the agricultural shares in income and employment with the level of income. They also find a lower labor productivity in agriculture than in manufacturing. This productivity gap even widens in early stages of development since – contrary to *Kuznets'* results – agricultural productivity growth lags behind that of the rest of the economy. Hence, it is not really clear which sector leads in terms of productivity increases in economic development. *Dixit* (1973, 328) summarizes and quantifies the most important stylized facts: (i) a decline in the proportion of agricultural labor from 70% to less than 20%, (ii) a decline in the share of agriculture in national product from around 50% to 15%, and (iii) an increase in labor productivity in both sectors. *Lewis* (1954) points out a fourth aspect, namely an increase of savings as a fraction of national income from 4-5% to 12-15%.

2.3.1 Dual Economy Models

These stylized facts of development gave rise to early theoretical models trying to replicate them: the dual economy literature. The dual economy models from the 1950s and 1960s are a sub-class of two sector models to analyze developing economies which are on the transition from a purely agrarian to an industrialized economy.[18] This special type of model permits the analysis of development aspects which disappear in any higher aggregation. The particular feature analyzed here is the coexistence of two sectors in an economy, agriculture and manufacturing or industry, which are basically asymmetrical, and thus dualistic, in terms of both product and organizational characteristics.[19]

The first asymmetry in dual economy models is caused by the special role of agricultural output, food. Consumption of food is essential for workers in both sectors alike. This does not apply, at least not generally, to the output of manufacturing. Hence agricultural and non-agricultural goods cannot be fully substituted for each other. In a closed economy this peculiarity of food implies a dependency of the manufacturing sector on agriculture. The converse, however, is not true.

The second asymmetry relates to the production techniques at early stages of industrialization. Agricultural production is characterized by diminishing returns since food is produced by (fixed) land and labor. If no technical progress exists in this sector, presence of the fixed factor land implies diminishing returns. Production in manufacturing is different. Land is of (almost) no importance. Output in this sector is produced by labor and capital under (usually) constant returns. Thus, capital accumulation to overcome diminishing returns to labor only takes place in manufacturing.

These asymmetries describe the core of dual economy models. Additional assumptions about production in both sectors are made which, however, vary from model to model. For example, it is often assumed that labor is paid its average product in agriculture but its marginal product in industry. This is equivalent to assuming that agricultural land is either owned by farmers or that labor can capture the implied rent on land for other reasons, for example, because it is common property. Especially the latter is often observed in developing countries. For the industrialized sector the assumption is equivalent to the existence of capitalists who reinvest their share of output.

18. The term "duality" has many different interpretations in the development literature: There exist discussions about dualism of agriculture and industry, sociological dualism of Western and non-Western economic organizations, technological dualism of fixed versus variable coefficient technologies and "North-South" models analyzing duality for the world as a whole. For details about the meanings mentioned and some more definitions of dualism see the surveys mentioned below.
19. This definition of duality follows *Ranis* (1988).

The mentioned stylized facts of economic development as well as additional phenomena have been discussed in a large number of dual economy models, which have been reviewed by *Dixit* (1973), *Kanbur* and *McIntosh* (1988), and also *Ranis* (1988).[20] The first of these "modern" dual economy models has been presented by *Lewis* (1954) and later extended by *Ranis* and *Fei* (1961).[21] In the tradition of classical economists, wages in this model approach subsistence consumption in the presence of *Malthus*ian population pressure on scarce land which is being cultivated with traditional technologies. Due to a traditional organization of this sector labor is abundant, and hidden unemployment exists. Since labor is in surplus supply, its marginal product in agriculture is zero. Therefore a withdrawal of labor does not decrease agricultural output. Development in this economy consists of reallocating surplus agricultural labor from agriculture into industry where formerly agricultural workers become productive members of the industrial labor force. At first they are employed at the same subsistence wage as in agriculture. This reallocation continues until the industrial labor supply curve begins to turn up, which happens as soon as the agricultural labor surplus has been exhausted.

The neoclassical model of a dual economy developed by *Jorgenson* (1961) and later extended by *Zarembka* (1970) questions the hypothesis of an agricultural labor surplus[22] and discusses the dependency of manufacturing on agricultural food production in a purely neoclassical framework. In this model competitive firms in manufacturing pay marginal productivity wages while agricultural wages equal the average product. Labor is freely mobile between both sectors and (almost) equates wages across sectors.[23] Within this kind of model the authors

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20. Dual economy models also make up a large part of *Ziesemer* (1987).
21. The are sometimes called "modern" because the idea of a dual economy can already be found in much earlier economic analyses. Duality is one of the central aspects of classical economics as well as of the physiocrats' work. See *Ranis* (1988). The quintessence of the classical notion of duality can be found in Eltis (1984).
22. This hypothesis has been intensely discussed during the 1960s. By now the dispute has been settled, at least for an important part of the developing world. "On one traditional question in the literature of agriculture and economic development there is now virtual unanimity in Africa. The marginal product of labor in agriculture is positive. Disguised unemployment is a concept no longer much employed in the African context since the evidence that it is peak season labor which is typically the operative constraint in African farming systems is by now overwhelming." (*Helleiner*, 1975, 28). Cf. also *World Bank* (1982).

discuss determinants of the economy's growth rate as well as the prerequisites for viability (growing real wages).

The migration decision is looked at more closely by *Harris* and *Todaro* (1970). In their model industrial firms pay wages above the market clearing level which leads to urban unemployment. In the rural agricultural sector the wage adjusts to clear the labor market. Workers migrate to urban areas since they equalize the agricultural wage with the expected industrial wage, that is, the higher wage rate in the urban area times the probability of actually finding employment there.

Numerous other studies, most of them based on the neoclassical model,[24] have extended the dual economy literature in other directions. *McIntosh* (1975), for example, discusses development when the rate of population growth is endogenous and determined by the distribution of population between the two sectors. *Bose* (1968) and *Dixit* (1971) discuss industrialization as a planning problem within a general equilibrium framework. *Chen* (1987) considers the consequences of uncertainty in agricultural production for the development of the dual economy.

2.3.2 Agriculture Driven Development

While these dual economy models are able to replicate the major stylized facts of economic development, they are ambiguous about the central question: what drives development? In the early years of development policy after the second world war the answer of mainstream economists and politicians was almost unanimous: industry. Industrialization, as rapid as possible, was regarded as the policy imperative. This did not leave much room for modernization of the agricultural sector, which, quite the opposite, could and should be squeezed on behalf of the

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23. There exists an endogenous wage differential between both sectors – justified by costs of migrating from agriculture to manufacturing – which prevents full equalization of wages.
24. There does exists a second strand of dual economy models based on *Kaldor's* (1975) informal reflections about industrialization in a two-sector economy with agriculture and industry (*Canning*, 1988; *Thirlwall*, 1986; *You*, 1994). It is still in debate, how much these models really differ from the classical dual economy literature, e.g., the *Lewis* (1954) model (*Dutt*, *1992*; *Thirlwall*, 1992).

more dynamic sectors. *Timmer* (1988) summarizes the main reasons for this popular opinion:

> No policy efforts on behalf of agriculture's own modernization were needed because the sector declined naturally. Most interpretations of the Lewis model (1954), especially the Fei-Ranis version (1964), which became the main teaching paradigms, *ignored the factors needed to modernize traditional agricultural sectors* so that they could play positive contributory roles in the development of the rest of the economy. The structuralist views of Prebisch (1950) about declining terms of trade for traditional products and the importance Hirschman (1950) attached to linkages to 'modern' economic activities further diminished any apparent rationale for actively investing in the modernization of agriculture itself. [...]
>
> [Agriculture] is the home of traditional people, ways, and living standards – the antithesis of what nation builders in developing countries envisioned for their society. Moreover, agriculture was thought to provide the only source of productivity that could be tapped to fuel the drive for modernization. Surplus labor, surplus savings, and surplus expenditures to buy the products of urban industry, and even surplus foreign exchange to buy the machines to make them, could be had from an uncomplaining agricultural sector. [...] The unique features of agriculture as a sector were simply not widely understood in the 1950s. Nor was it accepted that the development of a modern agriculture was necessary as a concomitant to development of the rest of the economy. (*Timmer,* 1988, 288-289, italics added)

Although *Timmer* emphasizes the 1950s as the heyday of industry-led development strategies, they have had proponents at all times.[25] However, these strategies have not remained unchallenged. All the time some economists have emphasized modernization of the agricultural sector as precondition for development. Their point of view is supported by economic historians' analysis of the development process in advanced countries, mainly in England. They point out that the industrial revolution started in countries that had already experienced substantial increases in agricultural productivity. Economies that had not done so (e.g., Tsarist Russia) could not hold on to industrialization (*Bairoch,* 1973; *Jones* 1967).

John (1967) describes the development in England at the beginning of the eighteenth century: Productivity increases in agriculture had lead to a rising supply of

25. Cf., e.g., *Kaldor* (1967) and *Williamson* (1989). *Mellor* (1989, 305) complains that "contemporary development theory has had little place for agriculture in growth."

agricultural output, especially of wheat. Wheat prices concurrently had fallen by more than 25 percent compared to 50 years before while the prices for other nutrients like meat and dairy products had roughly remained constant. Since the expenditure for bread constituted a considerable fraction of total household expenditure, this price decrease meant a real rise in per-capita income for the mainly rural working population. In response, demand from this broad class for other articles than necessities increased, namely for tee, sugar, tobacco, gin, printed calicoes, linen, pottery and glassware. Satisfaction of this demand in turn fostered development of the manufacturing sector since the high demand made large-scale production profitable.

This evidence manifested itself in the belief of some economists that "... the spectacular industrial evolution would not have been possible without the agricultural revolution that preceded it." (*Nurkse*, 1953, 52) *Rostow* has even stronger beliefs; for him "revolutionary changes in agriculture are an *essential* condition for successful take-off." (*Rostow*, 1960, 8, italics added).[26]

Most arguments in favor of agriculture-led development follow the historical examples and focus on the demand for domestically produced mass-consumption goods which could emanate from the traditional sector if just productivity and therefore income were high enough (*Adelman*, 1984). This view is supported by the fact that in most developing countries the agricultural sector is rather large in terms of the labor force fraction employed in it: so for example 65 percent in East Asia, 72 percent in South Asia and even 74 percent in Africa (*World Bank*, 1988).

The rising demand for consumption goods observed in England is only one source of development: While it might be the most important effect when agriculture is still very traditional, the adoption of more modern techniques will over time also increase the use of industrial output like machines and fertilizer as input factors in agricultural production. This point is emphasized by the *World Bank*

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26. However, it is sometimes argued that one cannot compare the development process of England or the United States from the eighteenth or nineteenth century with development problems of sub-Saharan countries since the "agricultural conditions in the currently developed countries before the beginning of the industrial revolution must have been very different from those of the under-developed countries of Asia and Africa today." (Bairoch 1975, 40 f.)

which adopted some arguments in favor of agriculture-led development during the 1980s:

> The continuing importance of agriculture in the economies of the developing countries is reflected in the association between the growth of agriculture and of the economy as a whole.
>
> [...] Expanding agricultural production through technological change and trade creates important demands for the outputs of other sectors, notably fertilizer, transportation, commercial services, and construction. At the same time, agricultural households are often the basic market for a wide range of consumer goods that loom large in the early stages of industrial development – textiles and clothing, processed foods, kerosene and vegetable oils, aluminum holloware, radios, bicycles, and construction material for home improvements. (*World Bank*, 1982, 44-45)

Academic studies and experience in some developing countries showed that improvements of agricultural productivity might be a worthwhile task; it turned out that differences in agricultural productivity among countries are enormous: *Hayami* and *Ruttan* (1985) found that two thirds of the difference in agricultural labor productivity can be accounted for by differences in technology. And these technologies can be improved as the "Green Revolution" of the 1960s in India, the introduction of new, high yielding seed varieties has shown: In 1960 India's wheat production was 11 million tons. By 1984 it had increased to 46 million tons (*Schultz*, 1988, 342).[27] This new judgement of agriculture is at least partially reflected in development policy. An example is the set-up of International Agricultural Research Centers (IARC) in which agricultural technologies are developed that are especially suited for the climatic and social conditions of the area where they are supposed to be adopted.[28]

For economic historians, development theorists, as well as for practitioners of development policy, improvements of agricultural productivity are an important topic as this section has shown. However, it has not yet found its way into the new formal and general models of growth and development discussed in the last section, except on rare occasions.[29] This is even more astonishing as there exists a large literature in development economics discussing determinants of agricultural

27. Although the Green revolution has brought substantial benefits to India, it also led to new problems; see, e.g., *Hazell* and *Ramasamy* (1991).

28. For an assessment see *Evenson* (1986).

productivity. Most of these, agricultural research, technology adoption, or human capital accumulation are also at the heart of the endogenous growth literature. Therefore, a promising approach should be to combine these insights about agriculture with larger models of developing countries like the dual economy models by using tools from the NGT. This would enable us to discuss the influence of microeconomic determinants of research or human capital accumulation on growth and structural change of a developing economy. The following section provides some starting points for such a combination.

2.4 Agricultural Productivity and the New Growth Theory

Most of the development literature looking more closely at possible forces behind productivity improvements in agriculture started in the 1960s. Before, such improvements were considered to be easily obtainable: developing countries simply had to adopt some of the highly productive techniques which have already been invented in the past several hundred years in the now industrialized countries. The blueprints for these techniques are (almost) freely available. However, as *Krueger* (1991) notes, experience and research alike show that the potential benefits from these available techniques are much smaller than this "free blueprint" notion would suggest: First of all, technologies are not independent of factor endowments in the countries where they have been invented. Many technologies in industrialized countries are suited to labor-scarce and capital-abundant economies but not to developing countries which in most cases are capital-scarce and labor-abundant. And secondly, technologies for agricultural production must be especially suited to climatic and soil conditions as well as to nutrition needs and customs in the developing countries.

Thus, the availability of *some* technology per se is seldom of much use. If a country wants to make use of these blueprints, usually a considerable amount of skills is necessary to convert them into usable production technologies. Therefore technology creation or technology adaptation can be seen as a precondition for productivity increases in agriculture. Hence, a country has to conduct agricultural

29. For example, in the *Murphy-Shleifer-Vishny* (1989b) model of distribution and growth. Cf. footnote 14.

research and development, be it innovative or adaptive, and it has to accumulate sufficient human capital in agriculture to adopt new technologies and to use them correctly. A closer look at characteristics and determinants of these activities should yield some insights why some countries increase their productivity in agriculture faster than others.

Two outstanding characteristics of agricultural technology creation and adoption are externalities and increasing returns. Both are also highlighted by the endogenous growth literature. *Schultz* (1988, 345) emphasizes increasing returns to technology on the micro level: "It is helpful to think of each increasing returns occurrence as an economic event. Most increasing returns are small, micro events, as in the case of a farmer's increase in corn yields made possible by hybrid seed." To him increasing returns from technology adoption are internal, they occur only in the moment of adoption. The new technique, when adopted, is again subject to diminishing returns.

However, increasing returns in agriculture are not confined to the micro level as presumed by *Schultz*. Actual materialization might happen at the micro level – a farmer uses a new hybrid seed for a slightly higher cost than the traditional one and is able to increase her output by more than the additional input cost her.[30] But this is only because she does not have to bear the cost for research and development of this new type of seed. If she had to, she probably would not engage in research since her expected pay-off from using this seed would be below the cost of developing it. On the macro level though, it would pay to engage in research for new seed types since new types only have to be developed once and can then be produced with roughly the same cost as the traditional seed by every farmer. This is due to the fact that only the development of the design for the new seed is costly, not so much the production of the seed itself. *Romer* (1990) introduced the term "design" for knowledge of this kind and points out that, although it is a factor of production, it differs in a crucial way from other factors: its usage in production is nonrival, which means that many producers can use the factor at the same time.

30. Women do most of the work in subsistence agriculture, up to 80%. Cf. *Todaro* (1992, 269).

Since everyone can use the same seed design, there exist large externalities in the development of new seed varieties. Indeed, there is considerable evidence of large differences between private and social returns to agricultural research. This has first been shown by *Griliches'* (1958) seminal work on hybrid corn in the United States, its research cost and social returns. Other empirical work, also for developing countries, followed and showed annual rates of return on research between 30 and over 100 percent (*Evenson*, 1988). The larger fraction of this return should be regarded as external since most of agricultural research is conducted in national or international government institutions[31] who only to a small extend engage in own production or otherwise try to make profits from new technologies. Instead, technologies are distributed freely or only for a nominal amount.[32] Therefore innovation-driven-growth models, where innovation is caused by profit-seeking private innovators, are not the appropriate model to explain agricultural technical progress in developing economies. They are better suited to explain growth in industrialized countries.

Another possibility to increase productivity, improvement of human capital, has also been propagated by *Schultz*. For him, human capital are "attributes of acquired population *quality*, which are valuable and can be augmented by appropriate investment" (*Schultz*, 1981, 21, italics added). The stock of acquired

31. There exist different views in the development literature upon what drives the *creation* of new techniques, supply-push forces like research institutions or demand-pull forces (*Rayner* and *Ingersent*, 1991; *Thirtle* and *Ruttan*, 1987). In the demand-pull or induced innovation view agents respond to economic forces like prices or factor scarcities and create the appropriate technologies. In underdeveloped countries this does not seem to be the rule, though, mainly due to institutional barriers like insufficient patent rights, lack of credit markets to finance research, or simply a lack of appropriate human capital. In the second, supply-push view technology creation is an exogenous process conducted in universities and research institutions. Some of the new technologies turn out to be useful and are adopted by farmers.
While these views seem to be contrary at first glance, reality contains both: research institutions will (or at least should) take into account factor scarcities in the target country or area, and farmers will be lead by economic considerations when making the decision whether to adopt a new technology or not.

32. Of course some of these returns could be internalized by requiring larger fees for revealing new techniques to farmers if sufficient property rights for ideas exist. But first of all this is not the case in many countries and secondly a free distribution is usually politically desired to increase rural incomes. There are also technical reasons, though: Especially in agricultural production there exist technology improvements that are almost unexcludable, e.g., different ways of ploughing, irrigating, planting crops, or feeding cattle.

human capital consists of abilities and information that have economic value. In agriculture this is especially the ability of farmers to modernize agricultural production, that is, the knowledge "to use land, labor, and capital efficiently in response to the production opportunities associated with agricultural modernization." (*ibid.*, 24) Therefore Schultz considers the improvement of farmers' human capital a prerequisite for "transforming traditional agriculture" (*Schultz*, 1964). A similar argument is brought by *Rosenzweig* (1982). He points out that human capital serves two needs: First of all it plays a managerial role. It may enhance or be associated with higher productivity of labor and land inputs. But secondly, it is also dynamically important: Human capital is necessary to find new possibly profitable innovations, judge their profitability in comparison to other technologies and put the new ideas into production. *Lockheed*, *Jamison*, and *Lau* (1980) survey empirical studies estimating these effects and conclude that – on average – four years of education increase farm productivity by 7.4%. The effects of education, so their second conclusion, were much more likely to be positive in modernizing agricultural environments than in traditional ones which supports *Schultz'* and *Rosenzweig's* assertion.

These reflections suggest that the analysis of agricultural productivity improvements with tools from the endogenous growth literature should be promising. After all, endogenizing technology creation or adoption is one of the basic themes of the NGT as is human capital accumulation. The endogenous growth literature also emphasizes the distinction between static and dynamic effects which appears in the development literature as the different purposes of human capital. Externalities are also emphasized by this literature.

A further topic for discussion are poverty or subsistence constraints which are due to the fact that farmers in developing countries very often earn only their subsistence consumption. This implies that their ability to save or to sacrifice in any other way present consumption to obtain a higher consumption in the future is very limited. Within exogenous growth models this only influences capital accumulation. Farmers are carried out of this subsistence misery by exogenous technical progress. However, if technical progress does not fall like manna from heaven but involves sacrificing food production and consumption by allocating time to research, technology adoption, or human capital accumulation, time which could otherwise been used to produce food, farmers might be in an underdevelopment trap. They are simply not able to spend resources for productivity improvements. In most cases credit financing is also not possible due to missing or incomplete capital markets. It is generally easier for large agricultural organizations to get credit financing than it is for small peasants.[33]

A related topic emanates from the fact that the agricultural sector produces a special good, food. If agents live at their subsistence, their productivity might be impaired by malnutrition. This "efficiency wage hypothesis" originated in the work of *Leibenstein* (1957) and *Mazumdar* (1959) and has recently found more interest, both theoretically (*Dasgupta*, 1993) as well as empirically (*Fogel*, 1994). While usually a relationship between nutrition and productivity in production is emphasized, the endogenous growth approach points to a possible connection between nutrition and productivity improvements. Malnutrition might impair the ability to learn or to conduct research. This dynamic relationship could be far more important than the static effect.

Additional characteristics of agriculture in developing countries could, in principle, also be incorporated into larger models by the use of endogenous growth techniques. The probably most important one is the behavior of farmers towards risk. Most farmers are highly risk-averse and prefer old-fashioned techniques with a low mean but also low variance over insecure techniques promising a higher yield.[34] The consequences of this attitude for growth and structural change could, for example, be discussed in a stochastic model which contains elements from the NGT describing the farmers' behavior. Since the analysis conducted in this study is in a perfect-foresight environment, though, we do not pursue this possibility here (Cf. *Chen*, 1987 for risk in a dual economy).

The analysis conducted in the following chapters is based on the hypothesis of agriculture led development presented in section 2.3.2 above. Agriculture is the main focus of the analysis while industry is not discussed in detail. This is the opposite approach of that chosen by most studies presented in section 2.2. It is

33. Cf., Binswanger, Balaramaiah, Bhende, and Kashirsagan (1985). Recently, the relationship between the financial system and economic growth has found new interest, in academics as well as in development policy. Cf., e.g., *King* and *Levine* (1993).
34. For a discussion of this behavior cf. *Schultz* (1964) and *Bliss* and *Stern* (1982).

justified by the large evidence in favor of the hypothesis of agriculture led development. Those extensions of the basic model that deal with endogenous technical progress are based on the ideas of human capital accumulation from *Lucas* (1988) and learning by doing from *Arrow* (1962). Both have been described in section 2.1. These simple models are better suited to describe agricultural technical progress in developing countries than the models of innovation driven growth as we have already argued earlier in this section. Therefore the subsequent analysis can be placed somewhere between the simple analysis of growth performance from section 2.2.1 and the structural change models from section 2.2.2

3. The Dual Economy Revisited

In this chapter we introduce the baseline model for our analysis of growth and development in a dual economy. It is descendant from the dual economy models developed around 1960. However, while the latter are mostly descriptive, the model presented here is a social planning exercise describing the optimal behavior of an economy. Although most economies in reality are probably not always on their optimal growth paths, it is also discussed whether the model is able to replicate important stylized facts of economic development. These stylized facts are: (i) a decline in the proportion of agricultural labor from 70% to less than 20% as quantified by *Dixit* (1973, 328), (ii) a duration of this process of roughly 100 years as observed by *Kuznets* (1966) for the United States and Japan, and (iii) an increase in labor productivity in both sectors.[1]

The model is extended in subsequent chapters by focussing either on specific characteristics of agricultural *production* in developing economies (the first asymmetry constituting duality) or on the unique role *consumption* of the agricultural output food plays for these countries (the second asymmetry).

In the first section the dual economy model is presented and its equilibrium properties are discussed. Special attention is given to the question whether multiple growth paths or multiple equilibria can occur. In section 2 the model's dynamic properties are discussed and compared to stylized facts of economic development. The first comparison is only qualitatively and based on properties that can be derived algebraically: steady-state behavior as well as the influence of certain policies on the long-run outcome. But, as *Dixit* (1970, 229) notes, "asymptotic properties of the dual economy model are not very interesting," since the dual character of an economy vanishes in the long-run. Therefore the transitional dynamics towards the steady-state are derived in a second step by numerical methods. This allows us to discuss the process of economic development depicted by the model's transitional dynamics. These might last for a rather long time and

1. In section 2.3 we have mentioned two further stylized facts characterizing development, namely a decrease in the share of agriculture in national product and an increase in savings as fraction of national income. However, since national income is not defined in the set-up chosen here, for reasons which will become clear below, those facts cannot be replicated within this model.

therefore be important for real-world issues. The necessary methodologies are presented when needed. Finally, the last section summarizes this chapter's results.

3.1 Exogenous Technical Progress in a Dual Economy

The basic model of a two-sector economy consists of a traditional sector (agriculture) and a modern sector (industry or manufacturing)[2]. The agricultural sector produces food and the manufacturing sector some other good which we call "widget" in the remainder. Since this model is also the foundation and some kind of benchmark model for the work in subsequent chapters, we keep it as simple as possible by assuming the rates of technical progress in both sectors to be given exogenously. In the first subsection we derive the model's steady-state and in the second subsection we analyze its stability and uniqueness. At first sight stability might appear to be a purely technical issue. However, as the NGT has shown, the occurrence of certain kinds of equilibria, namely multiple equilibria or multiple balanced growth paths can be made responsible for the internationally divergent growth experiences. We want to know whether these equilibria can occur in our two-sector model and can possibly be made responsible for different degrees of industrialization.

3.1.1 The Model

The economy consists of L identical individuals who can work either in the agricultural or in the manufacturing sector. Total labor force L grows with a constant exogenous rate λ. We thus abstract from sector-specific human capital or transaction costs of switching between the two sectors.[3] Because of these assumptions the problem can be modeled as if individuals spent a fraction n of their fixed working time in the traditional and the other fraction $(1 - n)$ in the modern sector.[4] The sectors are indicated by the subscripts A and M respectively. Food in the traditional sector is produced with a constant returns to scale production function by

2. We use these terms interchangeably.
3. Thus all human capital is general human capital in the sense of *Becker* (1975). *Matsuyama* (1991) analyzes a two-sector economy with heterogeneous agents. *Glomm* (1992) an economy where the individuals have to undergo a time of unproductive apprenticeship after migrating from agriculture to industry.

labor and land. Land is normalized to one so that the production function for this sector can be written as

$$Y_A = A(nL)^{\alpha}, \qquad 0 < \alpha < 1$$

where A is the state of technology in agriculture which grows with the constant exogenous rate ν and Y_A is the economy's food production. It is assumed that total agricultural output goes to labor.[5] Note that agriculture is assumed to be capital-less. Thus, saving does not take place in this sector, either. Hence labor is paid its average rather than its marginal product. Overall per-capita consumption of food can be expressed by

$$(3.1) \quad c_A = \frac{Y_A}{L} = \frac{1}{L}A(nL)^{\alpha}.$$

Manufacturing produces widgets by labor and capital with a Cobb-Douglas technology.[6] As usual the output can be either consumed or invested. Then investment – and thus the change of the capital stock since we neglect depreciation – is characterized by

$$(3.2) \quad \dot{K} = MK^{1-\alpha}((1-n)L)^{\alpha} - Lc_M$$

where c_M denotes per-capita consumption of widgets. As in the agricultural sector the state of technology in manufacturing, M, grows with a constant exogenous rate μ. Note that the output elasticity of labor is the same in both sectors. This assumption is not crucial for most of the results obtained below but simplifies the analysis.

4. This might seem a strong assumption if agriculture is equated with rural areas and manufacturing with urban areas. While this might be the prevalent tendency, people do indeed work part-time as small-scale farm operators and as employee in some other kind of occupation. *Rosenzweig* (1984), e.g., discusses an Indian study showing that over 65% of rural Indian farm operator households received wage or salary income, primarily because of the small size of Indian farms.
5. This excludes the existence of landlords and thereby simplifies the analysis.
6. Widgets could be anything which is produced primarily with labor and capital. Separation of traditional and advanced sector is not that easy in reality. Agriculture could also use capital, while there might be manufacturing production almost without capital. *Jorgenson* (1961, 311) points out that at time of his writing the advanced sector in South-East Asia could be identified with plantation agriculture, mining and the extraction of petroleum whereas the traditional sector included peasant agriculture and fishing. In Japan the advanced sector could be identified with heavy industry and the traditional sector with agriculture, small manufacturing and most construction.

So far these equations characterize the production side of the economy. To specify the demand for produced goods, there exist two possibilities: The first is to explicitly state the demand functions as has been done in the original dual economy models by *Jorgenson* (1961) and *Zarembka* (1970). However, to explicitly analyze the individual's misperceptions of externalities – as shall be done in the next chapter – it is more appropriate to start out from a utility function which includes per-capita consumption of both goods and let this function being maximized by a social planner. As usual, this leads to the same outcome as the behavior of utility maximizing individuals under the assumptions of perfect foresight and perfect competition as long as there are no externalities.

We will use a two-good CRRA utility function for instantaneous utility of each individual, where consumption of both goods enters in a Cobb-Douglas manner.

$$(3.3) \quad u(c_A, c_M) = \begin{cases} \dfrac{(c_A^{\gamma} c_M^{1-\gamma})^{1-\sigma}}{1-\sigma} & \text{for} \quad \sigma \neq 1 \\ \ln (c_A^{\gamma} c_M^{1-\gamma}) & \text{for} \quad \sigma = 1 \end{cases} \qquad \text{where} \quad 0 < \gamma < 1, \sigma > 0.$$

This function implies an elasticity of substitution between both goods of one. The intertemporal elasticity of substitution is $1 / \sigma$, the inverse of the Arrow-Pratt measure of relative risk aversion, which – being constant – gave the utility function its name.[7] It is a measure for the willingness and ability of consumers to shift consumption over time. Note that c_A and c_M are not completely substitutable.

The problem is set up as an optimal control problem.[8] A social planner has to choose a time path for c_A, c_M and n (the control variables)[9] which is optimal by maximizing utility over the whole period considered. Given these paths and a given stock of capital at $t = 0$, equation (3.2) implies a time path for the capital stock K. Thus, equation (3.2) has to be taken into account as a constraint when choosing time paths for the controls because capital accumulation in one period changes next-period output available for consumption. The time paths for L, M, and A are given exogenously. The second constraint in the problem is equation (3.1), the production function for food. Substituting (3.1) for c_A into the utility

7. For derivation of constant-relative-risk-aversion utility cf. *Azariadis* (1993, 179).
8. For an overview of this technique cf. *Chiang* (1992) or *Kamien* and *Schwartz* (1981).
9. The variable n is a bounded control since its value must be in the interval (0,1).

function reduces the control variables to c_M and n which simplifies the algebra. Formalized the problem can now be written as:[10]

$$\text{(3.4)} \quad \max_{n,\, c_M} \int_0^{\infty} L \frac{\left[\left(\frac{A}{L} (nL)^{\alpha} \right)^{\gamma} c_M^{1-\gamma} \right]^{1-\sigma}}{1-\sigma} e^{-\rho t} dt$$

$$\text{s.t.} \quad \dot{K} = MK^{1-\alpha} ((1-n)L)^{\alpha} - Lc_M$$

where ρ is the (positive) discount factor for future consumption and L, M, as well as A are functions of time. The familiar way to tackle the kind of problem given by (3.4) is to solve the so-called current-value Hamiltonian, where a shadow price θ is assigned to the capital accumulation constraint. This current-value Hamiltonian is:

$$\text{(3.5)} \quad H_c = L \frac{\left[\left(\frac{A}{L} (nL)^{\alpha} \right)^{\gamma} c_M^{1-\gamma} \right]^{1-\sigma}}{1-\sigma} + \theta \left(MK^{1-\alpha} ((1-n)L)^{\alpha} - Lc_M \right)$$

H_c is the sum of current-period utility and capital investment, the latter valued at the shadow price θ. This so-called co-state variable θ is also a function of time. It can be understood as the price or value of one unit of invested capital at time t in utility equivalents at time 0 if the economy is on the optimal path. An optimal allocation must maximize H_c at every point in time. This is the case if the following four solution equations are satisfied:

$$\text{(3.6)} \quad \frac{\partial H_c}{\partial c_M} = (1-\gamma) \left[\left(\frac{A}{L} (nL)^{\alpha} \right)^{\gamma} c_M^{1-\gamma} \right]^{1-\sigma} c_M^{-1} - \theta = 0$$

$$\text{(3.7)} \quad \frac{\partial H_c}{\partial n} = \alpha\gamma \left[\left(\frac{A}{L} (nL)^{\alpha} \right)^{\gamma} c_M^{1-\gamma} \right]^{1-\sigma} n^{-1} - \alpha\theta MK^{1-\alpha} (1-n)^{\alpha-1} L^{\alpha-1} = 0$$

$$\text{(3.8)} \quad \frac{\partial H_c}{\partial \theta} = \dot{K} = MK^{1-\alpha} ((1-n)L)^{\alpha} - Lc_M$$

$$\text{(3.9)} \quad \dot{\theta} = \theta\rho - \frac{\partial H_c}{\partial K} = \theta\rho - \theta(1-\alpha) MK^{-\alpha} ((1-n)L)^{\alpha}$$

10. In the following we do not explicitly state the special form of the utility function for $\sigma = 1$.

To be an optimal solution, the control variables c_M and n must be chosen in a way that satisfies the boundary conditions. These are (i) an initial value for capital stock K_0 and (ii) the transversality condition $\lim_{t \to \infty} e^{-\rho t}\theta K = 0$.[11] The solution must satisfy the sufficiency conditions as well to ensure that it is indeed a maximum and not a minimum. We show in appendix A.1 that *Mangasarian's* sufficiency conditions are always met for the assumed restrictions on the parameter values.

Within the system of equations (3.6) - (3.9) equation (3.6) describes the choice of c_M in every period which must be balanced such that the value of consuming a unit today must equal the value of saving it today and consuming the growth proceeds tomorrow. Equation (3.7) is a labor market condition. Labor will be allocated such that the marginal utility from working in the different productions is equal. Equation (3.8) simply assures that the optimal path is also feasible, and (3.9) delivers the rate of depreciation for the shadow price of capital θ. This variable decreases over time since investing at time 0 yields the highest gains by producing goods for the longest time; each machine put into use at a later point in time is producing for a shorter period.[12] The equations (3.6) - (3.9) together with the boundary conditions define the family of optimal paths. We consider only one of these, namely the steady-state equilibrium, that is, the equilibrium path on which all variables are either constant or grow with a constant though not necessarily equal growth rate. In the steady-state also the growth rate of the shadow price θ has to be constant and therefore from equation (3.9) the marginal product of capital (the real interest rate) is also a constant:

$$(3.10)\quad (1-\alpha)MK^{-\alpha}((1-n)L)^{\alpha} = \rho - \frac{\dot{\theta}}{\theta} = \text{const.}$$

---

11. This transversality condition is not universally accepted as a necessary condition for an infinite horizon problem. There exist several counter-examples in which the stated transversality condition is not met. Therefore *Michel* (1982) proposes to use $\lim_{t \to \infty} e^{-\rho t}H_c = 0$ instead.

 We follow *Barro* and *Sala-i-Martin* (1995, 508), who have pointed out that the above condition might well be used as a necessary condition since all known counter-examples involve no time discounting. In contrast all our problems do feature time-discounting as well as an objective function that converges.

12. For a more complete economic interpretation of the optimal control technique cf. *Dorfman* (1969).

Comparing this result with equation (3.8), we see that the growth rate of capital can only be constant if Lc_M / K remains unchanged since the first term on the right of (3.8) is simply the marginal product of capital divided by $(1 - \alpha)$. Therefore we have

$$(3.11)\quad \frac{\dot{c}_M}{c_M} = \frac{\dot{K}}{K} - \lambda.$$

Hence, as usual in neoclassical growth models, capital and consumption per capita (of industrial goods) grow with the same rate. Differentiation of equation (3.10) with respect to time (acknowledging that n as well as the growth rate of θ are constant in the steady-state) leads to

$$(3.12)\quad \mu - \alpha\frac{\dot{K}}{K} + \alpha\lambda = 0.$$

From combination of equations (3.11) and (3.12) we obtain the steady-state growth rates of capital and per capita consumption of industrial goods, which are denoted by a superscript star, as:

$$(3.13)\quad \left(\frac{\dot{K}}{K}\right)^* = \frac{\mu}{\alpha} + \lambda$$

$$(3.14)\quad \left(\frac{\dot{c}_M}{c_M}\right)^* = \frac{\mu}{\alpha}$$

Therefore – and this is the first result of this chapter – in the steady-state equilibrium the growth rate of per-capita consumption of industrial goods depends only on technical progress in this sector, not in any way on the outcome of agriculture.

We can observe as a second result a somewhat similar feature of the model in the agricultural sector. The growth rate of per-capita food consumption is simply given by differentiation of equation (3.1) with respect to time as

$$(3.15)\quad \left(\frac{\dot{c}_A}{c_A}\right)^* = \nu - (1 - \alpha)\,\lambda.$$

The growth rate of per-capita total consumption depends only on the rate of technical progress in agriculture ν and the population growth rate λ. Viability, that is, non-decreasing per-capita food consumption therefore requires $\nu \geq (1 - \alpha)\,\lambda$. The rate of technical progress in this sector must be large enough to counteract the Malthusian forces of a growing labor force and decreasing returns to labor in food production.

From comparing equation (3.14) to (3.15) one can also see that the ratio of food consumption to consumption of manufacturing goods generally is not constant, and that the question of which part of the basket expands depends on the parameter values. Therefore, the model is able to show a declining fraction of food in overall consumption over time. Note, that this does not even require strong assumptions. Even with similar rates of technical progress in both sectors, per capita consumption of widgets grows faster than per-capita consumption of food, and thus the share of manufactured goods will increase since $\alpha < 1$ and $\lambda > 0$. This behavior is very close to *Dixit*'s second stylized fact mentioned in section 2.3 and in footnote 1 above which, at the individual level, is also known as Engel's law.[13] However, it is not exactly the same since we have not derived a relative price between both goods and therefore total income is not defined in this model.

Recall that the system of equations (3.6) - (3.9) must satisfy the transversality condition $\lim_{t \to \infty} e^{-\rho t}\theta K = 0$ which has not been shown so far. This boundary condition can only be met if in the steady-state the product of θ and K grows with a rate smaller than the discount rate ρ. Differentiating (3.6) with respect to time, substituting the result into the condition to eliminate the growth rate of θ, and replacing the growth rates of c_M and K with their steady-state values, we get

$$(3.16)\quad \rho > \lambda + (1-\sigma)\left[(\alpha-1)\gamma\lambda + \gamma\nu + (1-\gamma)\frac{\mu}{\alpha}\right].$$

In the remainder we will assume this condition to hold.[14] It is necessary to ensure convergence of the integral in the optimal control problem. The discount rate must be larger than the growth rates of population and utility per capita.

So far the emphasis was only on the economy's growth behavior. Since this model contains two sectors, we can also consider its structure. This, outcome of the

13. Cf. section 6.1 for a detailed account of Engel's law.
14. Note that for $\sigma = 1$ this condition reduces to the simple requirement that the social discount rate must be larger than the growth rate of labor. This condition has already been pointed out by *Cass* (1965). Economically it simply means that the social planner values the utility of nearby generations more than that of future generations. Since per-capita utility is weighted by the labor force in the integral, ρ must be greater λ to discount at all.

development process, is characterized by n, the fraction of labor in agriculture. In the steady-state it has to be constant; it cannot increase or decrease forever at a constant rate. This steady-state value can be calculated by rearranging (3.8):

$$\frac{Lc_M}{K} = MK^{-\alpha}((1-n)L)^{\alpha} - \frac{\dot{K}}{K}$$

Also substitution of equation (3.6) into (3.7) to eliminate θ gives

$$\frac{Lc_M}{K}\frac{(1-n)}{n}\frac{\gamma}{(1-\gamma)} = MK^{-\alpha}((1-n)L)^{\alpha}.$$

Last, differentiating equation (3.6) with respect to time and substituting the result into equation (3.10) yields:

$$(1-\alpha)MK^{-\alpha}((1-n)L)^{\alpha} = \rho - (1-\sigma)\left[(\alpha-1)\gamma\lambda + \gamma\nu + (1-\gamma)\frac{\mu}{\alpha}\right] + \frac{\mu}{\alpha}$$

Combination of these three equations to eliminate Lc_M / K as well as the marginal product of capital leads after some rearrangements to the steady-state value for n,

$$(3.17)\quad n^* = \frac{\gamma\left(\rho - (1-\alpha)(1-(1-\sigma)\gamma)\lambda - (1-\sigma)\left(\gamma\nu + (1-\gamma)\frac{\mu}{\alpha}\right) + \mu\right)}{\rho - (1-\alpha)\sigma\gamma\lambda - (1-\sigma)\gamma\nu + \sigma(1-\gamma)\frac{\mu}{\alpha} + \gamma\mu}$$

which is positive and smaller one by the transversality condition as is shown in appendix A.2.

Obviously, all parameters of the model influence n^*. This opens the possibility for economic policy to influence not only the output growth rate in both sectors but also its steady-state structure, that is, to industrialize a country. We will analyze the effects of economic policy modeled by parameter changes on the economy's steady-state structure in section 3.2.1.

3.1.2 Steady-State Uniqueness and Stability

We next analyze the local uniqueness and stability properties of the model's steady-state solution. The necessity to discuss these properties in this kind of optimal control problem has recently been emphasized in the endogenous growth and the real business cycle literature.[15] The dynamics of models only slightly more complicate than the simple neoclassical one-sector growth model can become rather complicate, and equilibria might be unstable or not unique. If the model shows multiple steady-state growth paths or multiple equilibria, this can have important economic implications, especially for a cross-country comparison of growth and development and thus for explanations of different growth performances.

Multiple steady-state *growth paths* can occur independently of the model's local stability around the steady-state equilibrium. Consider, for example, a set of countries where all parameters describing preferences and production are equal. All countries grow with the same growth rates in the steady-state and have the same division of labor between sectors. If multiple steady-state growth paths exist, their *levels* of production and consumption, though, can be completely different, depending on the initial values for the state variables. These multiple growth paths usually occur when the steady-state equilibrium requires a fraction of two state variables to be constant. In this case there exists an infinitely large number of fractions fulfilling this condition. Only for equal initial conditions do countries converge to the same steady-state growth path. Multiple growth paths thus imply that history matters. If there are, as in *Lucas* (1988), two stocks of capital (physical and human), and equilibrium requires a constant relation of those, then an economy with low starting values of both converges to a steady-state with lower growth paths than an economy with higher stocks of capital at the outset. The less endowed country will never catch up in levels with better endowed countries.

Independent of the existence of multiple growth paths is the question, whether the steady-state equilibrium is stable. The local stability analysis around the steady-state can lead to three results: the steady-state might be unique and stable, unique and unstable, or it might be indeterminate. The last case is also referred to as existence of *multiple equilibria.* Uniqueness is generally the desired result while instability is undesirable since it implies that an economy that has once left its steady-state equilibrium will never return to it. Since in reality economies are fre-

15. See especially the papers presented on a conference on growth, fluctuations, and sunspots which have subsequently been published in the *Journal of Economic Theory.* A summary is given by *Benhabib* and *Rustichini* (1994).

quently exposed to shocks, such an outcome would make the model even more unrealistic as it already is.

It is not really well understood, when indeterminacies arise. Externalities seem to be a necessary but not a sufficient condition. *Boldrin* and *Rustichini* (1994) show that in simple one-sector models equilibria are usually unique, while for a two-sector economy with a capital good sector and a consumption good sector multiple equilibria are present under rather mild assumptions. *Benhabib* and *Farmer* (1994), on the other hand, present a one-sector model where utility is separable in consumption and leisure and show that indeterminacies are possible for realistic parameter values.

If the equilibrium is indeterminate and thus multiple equilibria exist, even identically endowed economies can converge to different paths of output and capital in the steady-state. As in a unique equilibrium the economies converge to the same steady-state growth rates. However, they might have chosen different time paths for their control variables during the transition period towards the equilibrium.

> Therefore it is possible to view cultural and non-economic factors in such models not as affecting fundamentals like technology or preferences, but simply as a selection device for equilibria which differ on the transition paths. (*Benhabib* and *Perli*, 1994, 116)

Under certain circumstances multiple equilibria may lead to the possibility that a country with lower endowments can choose its control variables in such a way that it overtakes an initially richer country (*Xie*, 1994). The question remains, though, what in this case determines a country's choice of control variables like consumption. The answer then can only be found outside the model that solely shows the possibilities a country has. History, expectations, as well as the economy's structure can play an important role in the choice process (*Krugman*, 1991).

To answer the question, whether multiple growth paths or multiple equilibria exist, we start by transforming the solution for the control problem, here the dynamic system given by (3.6) - (3.9), into a system of differential variables that remain stationary in the steady-state. Basically this is the same as introducing the variable "augmented labor" into traditional neoclassical growth models. Since we have three observable endogenous variables (c_M, K, and n), we also need three stationary variables. The first, n, is already given: the division of labor between the two sectors must be constant in the steady-state. Natural candidates for the second and third are given by equations (3.10) and (3.11). Equation (3.11) implies that the term $L\, c_M / K$ be constant in the steady-state. (Recall that this is how we have derived the equation.) Therefore, we define a new variable $z_1 = Lc_M / K$ and – following *Mulligan* and *Sala-i-Martin* (1993) – call it a control-like variable since it contains the control c_M. The last variable can be obtained from equation (3.10) which requires that $z_2 = ML^{\alpha} / K^{\alpha}$ be stationary. We call this new variable state-like.

This variable transformation already answers the question about the existence of multiple steady-state growth paths. The new state-like variable z_2 contains a fraction of two state-variables, ML^{α} / K^{α}. However, contrary to *Lucas'* model only one of those is endogenous. Therefore endogenously determined multiple equilibria are not possible. There do exist, however, different growth paths if the state of technology in industry, M, is not equal for all countries. This might be the case if the flow of knowledge and technology (which is usually the intuitive interpretation of M) is imperfect: suppose that M in the developing country remains permanently below the state of technical knowledge in a more advanced country. Suppose further that both stocks of knowledge grow with the same rate and that all other parameters are the same. Then both economies have the same steady-state values for z_2. Thus, K must grow on a path permanently below that of an advanced country. Therefore, also its growth path of per-capita widget consumption is lower. Note that the growth path of per-capita food consumption, as well as the steady-state structure of the economy remain unaffected, though. Hence, similar effects for the growth path of food consumption require barriers to the flow of agricultural technology.[16]

We now consider the local stability and the question whether multiple equilibria are possible, in two steps. In the first step, we set $\sigma = 1$ and analyze the stability properties algebraically. This gives a clear result at least for this special case. For the more general situation, however, the equations quickly become too complex

16. We have argued in chapter 2 that there exist differences in usable agricultural techniques due to different soils, climate, etc., so that these barriers indeed exist. Cf. also chapter 5.

for algebraic discussion. Therefore we discuss the stability properties for a larger plausible range of parameter values numerically.

Before deriving differential equations for three variables, we can simplify the analysis. Substituting equation (3.6) into (3.7) to eliminate θ and using the definitions for z_1 and z_2 leads after some rearrangements to

$$z_1 = \frac{(1-\gamma)}{\gamma} z_2 n (1-n)^{\alpha - 1}. \tag{3.18}$$

Hence, if z_2 and n are known, at every moment in time is z_1 given by equation (3.18). Therefore the model can be reduced by one variable and stability can be analyzed from differential equations for z_2 and n.

The former can be obtained by differentiation of the definition for z_2, use of (3.8) as well as the definitions for z_1 and z_2 as

$$\dot{z}_2 = z_2 (\mu + \alpha\lambda - \alpha z_2 (1-n)^{\alpha - 1} (\frac{\gamma - n}{\gamma})). \tag{3.19}$$

Differentiating equation (3.18) with respect to time and substituting the growth rates of z_1 and z_2 from the variable definitions and the growth rate of c_M from the combination of equations (3.6) and (3.9) into the result yields a differential equation for the change of n:

$$\begin{aligned} \dot{n} = {} & \frac{n(1-n)\,[\sigma\lambda(1-\alpha) - \mu - \rho + (1-\sigma)\,(\mu(1-\gamma) + \gamma\nu)\,]}{\sigma(1-\alpha n) + (1-\sigma)\gamma(1-\alpha)} \\ & + \frac{(1-\alpha)\,(1-\gamma)\,z_2 n (1-n)^{\alpha} \dfrac{(\sigma n + \gamma(1-\sigma))}{\gamma}}{\sigma(1-\alpha n) + (1-\sigma)\gamma(1-\alpha)} \end{aligned} \tag{3.20}$$

This rather complicate differential equation simplifies for $\sigma = 1$ to

$$\dot{n} = \frac{n(1-n)}{(1-\alpha n)} \left[\lambda(1-\alpha) - \mu - \rho + (1-\alpha) z_2 n (1-n)^{\alpha - 1} \frac{(1-\gamma)}{\gamma}\right]. \tag{3.21}$$

For the algebraic discussion we consider only equations (3.19) and (3.21). The steady-state values for n and z_2 are derived by setting those equations equal to zero and solving for z_2 and n. There exist three solutions, of which one is interior and the others are corner solutions. The interior solution is given by:

$$(3.22)\quad n^* = \frac{\gamma(\rho-(1-\alpha)\lambda+\mu)}{\rho+(1-\gamma+\alpha\gamma)\frac{\mu}{\alpha}-(1-\alpha)\gamma\lambda}$$

$$(3.23)\quad z_2^* = \frac{(\rho+\frac{\mu}{\alpha})}{(1-\alpha)}\left[\frac{(1-\gamma)(\rho+\frac{\mu}{\alpha})}{\rho+(1-\gamma+\alpha\gamma)\frac{\mu}{\alpha}-(1-\alpha)\gamma\lambda}\right]^{-\alpha}$$

Note that equation (3.22) is identical to (3.17) with $\sigma = 1$. The second solution is given by $n = 0$ and $z_2 = \lambda + \mu / \alpha$. In this case the primary sector has disappeared. In the third solution, $n = 1$ and $z_2 = 0$, the secondary sector vanishes. In both cases the dual economy collapses. In addition both corner solutions violate the transversality condition (cf. appendix A.2), so that only the interior solution remains.

The equilibrium described by equations (3.19) and (3.21) - (3.23) is unique and stable (the steady-state growth path is determinate) if the system's Jacobian evaluated at the steady-state has one eigenvalue with positive and one with negative real part. This stability condition for an optimal control problem is different from not-controlled systems that require negative real parts for both eigenvalues to be stable. The reason is that one of the two variables, *n*, is a control variable and can be controlled to lead to an equilibrium.[17] We obtain the Jacobian J^* from equations (3.19) and (3.21) and evaluate it at the steady-state given by equations (3.22) and (3.23). Its eigenvalues are given by the solution to its characteristic equation

$$r^2 - \mathrm{Tr}J^*\, r + \mathrm{Det}J^* = 0$$

where $\mathrm{Tr}J^*$ is the trace of the evaluated Jacobian J^* and $\mathrm{Det}J^*$ its determinant. Instead of calculating the eigenvalues by solving the characteristic equation – which yields rather complicate expressions – we can make use of the Routh-Hurwitz theorem.[18] It states that the number of roots with positive real parts is equal

17. For a more extensive discussion of this topic cf. *Lorenz* (1993). The other two possibilities here are an unstable equilibrium (two positive real parts) or stability with multiple equilibria (two negative real parts).
18. The familiar Routh-Hurwitz *conditions* are a special case of this theorem. Cf., e.g., *Gantmacher* (1966).

to the number of sign variations in the following scheme (see also *Benhabib* and *Perli*, 1994, Theorem 1):

$$1, \quad -\mathrm{Tr}J^*, \quad \mathrm{Det}J^*$$

Determinant and trace of the Jacobian can be obtained as:

$$(3.24)\quad \mathrm{Det}J^* = -\frac{\alpha\left[\rho+\frac{\mu}{\alpha}\right]\left[\rho+\mu-(1-\alpha)\lambda\right]\left[\rho+(1-\gamma+\alpha\gamma)\frac{\mu}{\alpha}-(1-\alpha)\gamma\lambda\right]}{(1-\alpha)\left[(1-\gamma+\alpha\gamma(1-\alpha))\left(\frac{\mu}{\alpha}-\lambda\right)+(\rho-\lambda)(1-\alpha\gamma)\right]}$$

$$(3.25)\quad \mathrm{Tr}J^* = \rho-\lambda$$

Since the transversality condition (3.16) reduces to $\rho - \lambda > 0$ for logarithmic utility ($\sigma = 1$), it is immediately obvious that the trace of J^* is strictly positive. In this case there can be either one change of sign in the above scheme (+, –, –) and therefore only one eigenvalue with positive real part or two sign changes with two eigenvalues with positive real parts (+, –, +). In the first case the equilibrium is locally saddle-path stable and unique, in the second it is unstable.

Hence, everything depends on the sign of $\mathrm{Det}J^*$. By the assumptions about α, γ, and by the transversality condition all brackets in the numerator are positive. The second, bracketed term in the denominator is also positive by the transversality condition. Therefore $\mathrm{Det}J^* < 0$ and the equilibrium is locally saddle path stable and unique.

For the more general case of non-logarithmic utility no simple analytical solutions for the conditions derived from the Routh-Hurwitz theorem can be found. Since this case is, however, the more realistic one (cf. section 3.2.1) we have conducted numerical calculations to check the above result for more general situations. For this purpose the Jacobian is calculated numerically from equations (3.19) and (3.20) for a range of parameter values. The eigenvalues can be obtained numerically from this matrix using *Mathematica's* Eigenvalue routine[19]. We have chosen $\alpha = 0.7$ and $\rho = 0.05$, both conventional values. Dixit (1973, 332) notes that for agriculture $\alpha = 0.6$ would be in rough agreement with labor share data from India and Japan. The value $\alpha = 0.7$ is also within the range reported by *Maddison* (1987, table 8) for industrialized countries. For λ, μ, ν, and γ a low and a high

19. See *Wolfram* (1991).

value are chosen to obtain results for a broad range. The variable σ is varied over the range from 0.1 to 10. The results are given in table 1.

Table 1: Saddle Path Stability for Baseline Model

γ	λ	μ	ν	σ	γ	λ	μ	ν	σ
0.8	0.02	0	$(1-\alpha)\lambda$	0.1 – 10	0.4	0.02	0	$(1-\alpha)\lambda$	0.1 - 10
			0.02	0.1 – 10				0.02	0.1 - 10
		0.02	$(1-\alpha)\lambda$	0.1 – 10			0.02	$(1-\alpha)\lambda$	0.1 - 10
			0.02	0.1 - 10				0.02	0.2 - 10[a]
	0	0	$(1-\alpha)\lambda$	0.1 - 10		0	0	$(1-\alpha)\lambda$	0.1 - 10
			0.02	0.1 - 10				0.02	0.1 - 10
		0.02	$(1-\alpha)\lambda$	0.1 - 10			0.02	$(1-\alpha)\lambda$	0.1 - 10
			0.02	0.1 - 10				0.02	0.1 - 10

a. $\sigma = 0.1$ violates transversality condition (3.16).

Table 1 shows that the basic dual economy model is saddle path stable for a broad range of parameter values, not only for the special case of $\sigma = 1$. We can therefore quite safely rule out the possibility of multiple equilibria for reasonable parameters. Note that this does not mean that multiple equilibria are impossible.

3.2 Economic Policies, Preferences, and Development

Having derived the model's steady-state, we can now analyze the economy's reaction to economic policies or to changes in preferences. We can also compare its adherence to the three stylized facts of economic development mentioned in the introduction to this chapter. This is done in two steps: First, we conduct a comparative static analysis by deriving the effects of different kinds of economic policy on steady-state growth rates and on the economy's long-run structure. Structural change will be understood as a change of the fraction of labor employed in each sector, development as an increase of the labor fraction in industry together with a non-decreasing per-capita consumption of both goods. Within the development literature, increases in rates of technical progress as well as a decrease of the population growth rate are often mentioned as examples for policies that foster economic development. But also changes in preferences might have an influence on the economy's steady-state. We will check the validity of these assertions for our model. The results are then confronted with the stylized facts.

In the second step we actually consider the *process* of structural change by studying the model's transitional behavior. This yields some insights about the short-run dynamics of the economy and the time-scale of economic development. We would like to know how changes in preferences or economic policies can influence the economy in the short-run. Again we confront the model's implications with empirical evidence. In both steps numerical calculations are conducted. The first subsection discusses the long-run dynamics and the second discusses the short-run implications of the model.

Combining numerical and algebraic methods, which so far has not been done very often, has two advantages over either one of these alone. Compared to a purely algebraic discussion, it enables us to discuss also transitional economics and quantitative properties of the model. Compared to purely numerical analysis it reduces the risk of arbitrary results since at least some properties of the model can be derived algebraically. In our case these are steady-state behavior and stability properties for a simplified utility function. Also some of the dynamics within the model are known from the algebraic results. This reduces the risk of misinterpreting model assumptions as the result of endogenous model dynamics.[20]

3.2.1 Long-Run Effects

There exist several economic policies which might, from the first impression, be favorable to industrialization of a country. These are, e.g., increasing the rate of technical progress in agriculture and industry, or decreasing the population growth rate. Within our model such policies can have three kinds of effects. First of all, they can change the steady-state growth rates of some variables; of interest are usually per-capita consumption of both goods. Secondly, the policies can change the economy's structure, given by the division of labor between the sec-

20. An infamous example is *Meadow's et al.* (1992) study of "Limits to Growth". As *Nordhaus* (1992) has shown, the model has incorporated limits to growth in several independent assumptions, each alone would suffice to let growth come to an end eventually. Nevertheless *Meadows et al.* claim that this prediction is the *result* of the model dynamics.

tors. And thirdly, they can change the levels of growth paths for consumption of food or widgets. But not only changes of λ, ν, and μ influence the steady-state. Also changes in preferences, in γ or σ might influence the economy's structure. They do not, however, influence the consumption growth rates, as the respective equations derived above show.

In section 3.1.1 we have already stated or derived some of the necessary formulas for the policy and preference analysis. These are: the production functions for agricultural (3.1) and manufacturing goods (3.2), the steady-state growth rates of both goods given by equations (3.14) and (3.15), as well as the economy's steady-state structure given by the fraction of labor employed in agriculture and stated in equation (3.17). While the first four equations are rather simple, the effects of changes in μ, ν, λ, σ, or γ upon n^* cannot be seen directly from equation (3.17). We can, however, show by taking derivatives of equation (3.17) in appendix A.3 that the following conditions hold (no equally simple statements can be made about $\partial n / \partial \gamma$):

Table 2: Derivatives of n^*

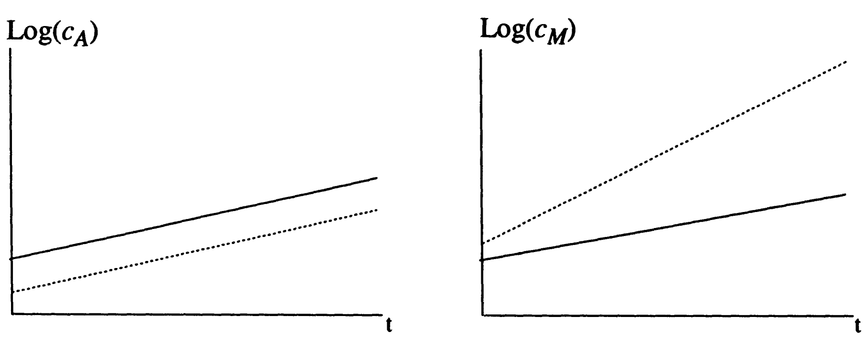
Derivatives	$\sigma < 1$	$\sigma = 1$	$\sigma > 1$
$\partial n^* / \partial \mu, \lambda > 0$	(–)	(–)	(?)
$\partial n^* / \partial \mu, \lambda = 0$	(–)	(–)	(–)
$\partial n^* / \partial \nu, \mu + \alpha\lambda > 0$	(–)	0	(+)
$\partial n^* / \partial \nu, \mu + \alpha\lambda = 0$	0	0	0
$\partial n^* / \partial \lambda$	(–)	(–)	(–)
$\partial n^* / \partial \sigma, \mu + \alpha\lambda > 0$ and/or $\nu > (1-\alpha)\lambda$	(+)	(+)	(+)
$\partial n^* / \partial \sigma, \mu + \alpha\lambda = 0$ or $\mu = 0, \nu = (1-\alpha)\lambda$.	0	0	0

Table 2 shows that almost all signs are identical for different assumptions about σ. Only the influence of agricultural technical progress on the economy's structure depends on this parameter. Empirical evidence, however, suggests that σ might be considerably larger than unity. *Hall* (1988), for example, has estimated values around 10 for σ. *Giovannini* (1985) has obtained similar low values for the intertemporal elasticity of substitution, in some estimations not even significantly different from zero. Therefore, unless explicitly stated otherwise, we will assume $\sigma > 1$ for the remainder.

Consider first an increase in the rate of technical progress in industry, μ The growth rate of consumption for manufacturing goods rises while the growth rate of food consumption remains unchanged. The effect upon n^* and thus on the level of the food consumption path is negative as long as λ is not too large. Otherwise it might be positive if $\sigma > 1$. We consider only the first case here. The second case is just the opposite. An increase in μ shifts labor from agriculture to manufacturing, thereby decreasing the level of food consumption and increasing the level of widget consumption.

Figure 1 shows that also in this model increasing the rate of technical progress in industry is a policy which fosters industrialization.[21] From the point of view of cross-country comparisons the model implies that economies with higher rates of technical progress in industry should be characterized by a (relatively) larger industrial sector and a lower level of food consumption. Although technical progress has a positive effect on per-capita consumption of widgets due to growth and level effects in the industrial sector, this kind of development is bought by a permanently lower food consumption path since a smaller labor fraction works in agriculture. One could argue, however, that this negative effect is not very important in the long-run.

Figure 1: Increasing Technical Progress in Industry



Legend: solid line = low μ; dotted line = high μ.

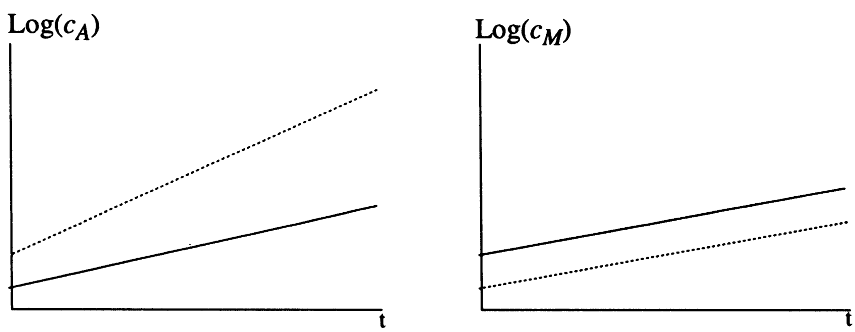
21. In the subsequent figures the transitional dynamics are neglected and only the long-run shifts of growth paths are shown. The figures are based upon table 2 and the solutions for the steady-state growth rate.

The forces behind this development can be derived from the solution to the optimal control problem, equations (3.6) - (3.9). Equation (3.7) in growth rates demands that the marginal utilities from working in either sector must grow with equal rates. By increasing μ the right-hand side of this equation rises, which can be rebalanced by a smaller growth rate of θ. According to equation (3.9) this can be achieved by decreasing n and employing a larger fraction of labor in manufacturing.

Such a development strategy could be named "industry pull strategy" or industry led development. Indeed, as we have discussed in section 2.3.2, this has been the major development strategy of the 1950s. While our exogenous growth model supports this strategy, it also points to a possible problem, namely a decrease in the level of food consumption. For countries close to subsistence consumption this might pose a serious problem.

Next consider a decrease in λ, the rate of population growth. While the growth rate of per-capita food consumption increases, the growth rate of manufacturing goods remains unchanged. The decrease in the population growth rate will also lead to a larger steady-state fraction of labor in agriculture[22] which in addition to the favorable growth effect on food consumption induces a level effect. Both are summarized in Figure 2.

Figure 2: Decreasing the Population Growth Rate



Legend: solid line = high λ; dotted line = low λ.

22. While n is larger in the steady-state, total labor in agriculture, nL, will in the beginning be more but eventually less than in a situation with higher λ. In manufacturing, total labor will always be less than in the high-λ case.

We observe a labor shift into agriculture and therefore higher levels of food consumption per capita (accompanied by a higher growth rate) and lower levels of widget consumption. The policy leads to a less industrialized economy. In cross-country comparisons one should therefore observe that countries with large population growth rates are highly industrialized. The reality, however, shows just the opposite. An explanation for this gap between model and reality might be the full-employment assumption of our (neoclassical) model. If unemployment were allowed to exist in the model, the appropriate variable would be *work force* growth and not population growth. With this variable the picture from reality is less clear. The outcome is also contrary to the outcome of traditional dual economy models. The reason for this difference is that in our model all additionally produced food is consumed while the existence of Engel effects in traditional models leads to a saturation of food consumption. Such effects also increase widget demand which drags labor into widget production.[23]

The main force behind the larger fraction of labor in agriculture comes from equations (3.8) and (3.13). Since the steady-state capital growth rate due to equation (3.13) is smaller the lower λ, less labor has to engage in capital accumulation and thus the fraction of labor in agriculture according to equation (3.8) becomes larger.

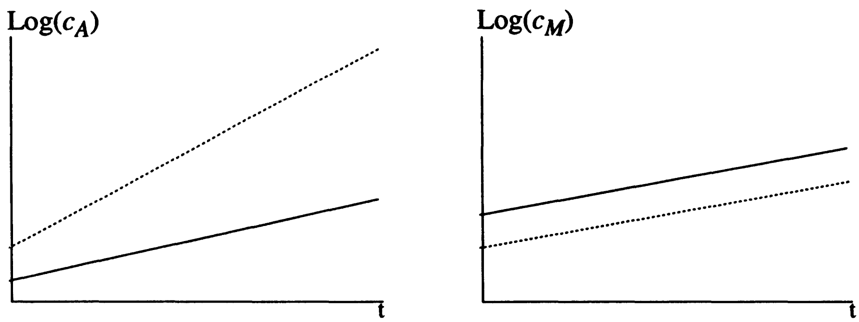
Next consider an increase in the rate of technical progress in agriculture, ν. According to equations (3.14) and (3.15) this will increase the growth rate of per-capita consumption of food and leave unchanged the growth rate of per-capita consumption of widgets. From table 2 we know that labor will shift towards agriculture (figure 3). Thus, economies with high rates of technical progress in agriculture should be characterized by a low degree of industrialization. Again, this outcome is contrary to the commonly observed facts.

Contrary to the last two derivatives neither equation (3.6) nor the labor-market condition (3.7) delivers any reasonable intuition for this outcome. This leads us to the conclusion that the long-run effect on n is not particularly stable and could vanish in a different set-up of the problem. This possibility is supported by the dependency of the effect on σ. Also appendix A.3 shows that the effect only exists

23. See chapter 6 for this effect.

for positive μ and/or λ. If both are zero, a change in ν has no effect whatsoever on the economy's steady-state structure.

Figure 3: Increasing Technical Progress in Agriculture



Legend: solid line = low μ; dotted line = high μ.

Last consider a change in σ. One would argue that this parameter cannot be easily influenced by economic policy since it describes the individual preferences. Nevertheless, other factors could be responsible for a change. Such a change would, however, only influence the economy's structure, not its growth rates. The derivatives show that the larger σ, the less industrialized is the country. These are economies with a low intertemporal elasticity of substitution ($1 / \sigma$) whose preferences conflict with easy consumption shifts. Since these are mostly poor countries, this outcome is in accordance with our first stylized fact. However, similar to the influence of ν, the effect seems to be rather fragile since it disappears for $\mu = \lambda = 0$.

So far these results describe the qualitative effects of preference changes and economic policies on the steady-state outcome. While this discussion has so far given some insights into the direction of these effects, these discussions alone do not tell us much about their strengths. We therefore calculate as a next step a series of steady-states for different parameterizations numerically. Besides giving information about the quantitative implications of economic policies these calculations can also give some insights about the causes behind the stylized facts mentioned in the introduction to this chapter. Such a question would be, for example, whether an increase in the rate of technical progress in manufacturing from zero to some positive, reasonable number is able to induce a decrease of agricultural labor from 70% to less than 20%. Finally, the calculations also yield the effects of changes in γ, the weight of food consumption in utility, which could not be obtained algebraically.

The equations for the steady-states of n and z_2 have already been derived as (3.22) and (3.23). The steady-state levels of food as well as of widget consumption growth, which are given by equations (3.15) and (3.11), are also reported. To be able to calculate steady-state values numerically, some additional assumptions about the parameter values of α and ρ have to be made: we choose again $\alpha = 0.7$ and $\rho = 0.05$. For each of the remaining parameters γ, λ, μ, and ν a low and a high value are employed giving a total of 16 steady-states. Again, the parameter values are conventional, $\nu = (1-\alpha)\lambda$ just satisfies the viability condition. The results for different values of σ (10, 5, and 1) are given in tables 3 - 5 below. Generally, the calculated values are of a realistic size. The marginal product of capital represents an exception, as it amounts to 30% for certain parameterizations.

Table 3: Steady-States for Different Parameter Values, $\sigma = 10$

γ	λ	μ	ν	n^*	$z_2{}^*$	$(\dot{c}_A/c_A)^*$	$(\dot{c}_M/c_M)^*$	r^a
0.8	0.02	0	(1-α)λ	0.779	0.479	0	0	0.05
			0.02	0.793	1.516	0.014	0	0.151
		0.02	(1-α)λ	0.780	1.252	0	0.029	0.13
			0.02	0.789	2.289	0.014	0.029	0.231
	0	0	(1-α)λ	0.8	0.514	0	0	0.05
			0.02	0.8	1.995	0.02	0	0.194
		0.02	(1-α)λ	0.789	1.287	0	0.029	0.13
			0.02	0.795	2.768	0.02	0.029	0.274
0.4	0.02	0	(1-α)λ	0.370	0.230	0	0	0.05
			0.02	0.385	0.470	0.014	0	0.100
		0.02	(1-α)λ	0.385	1.090	0	0.029	0.233
			0.02	0.387	1.331	0.014	0.029	0.283
	0	0	(1-α)λ	0.4	0.238	0	0	0.05
			0.02	0.4	0.581	0.02	0	0.122
		0.02	(1-α)λ	0.391	1.098	0	0.029	0.233
			0.02	0.393	1.442	0.02	0.029	0.305

a. The interest rate r is given by the marginal product of capital, $(1-\alpha)MK^{-\alpha}((1-n)L)^{\alpha}$.

Tables 3 - 5 show that the quantitative effects of economic policies like increases in the rate of agricultural technical progress are rather small. From the first eight rows in each table one can see that no reasonable shock on the production side of the economy changing either λ, μ, or ν is strong enough to induce a labor shift of the size in our first stylized fact.[24] While there is some change in n^* in the direction already derived above, this change is only small and cannot explain the observed phenomenon. Thus, changes in the production parameters alone cannot account for the most obvious stylized fact of economic development. Neither do changes in these parameters seem to be powerful economic policies for industrialization.

Table 4: Steady-States for Different Parameter Values, σ = 5

γ	λ	μ	ν	n^*	z_2^*	$(\dot{c}_A/c_A)^*$	$(\dot{c}_M/c_M)^*$	r^a
0.8	0.02	0	$(1-\alpha)\lambda$	0.779	0.479	0	0	0.05
			0.02	0.789	0.940	0.014	0	0.095
		0.02	$(1-\alpha)\lambda$	0.774	0.958	0	0.029	0.101
			0.02	0.783	1.419	0.014	0.029	0.146
	0	0	$(1-\alpha)\lambda$	0.8	0.514	0	0	0.05
			0.02	0.8	1.172	0.02	0	0.114
		0.02	$(1-\alpha)\lambda$	0.785	0.993	0	0.029	0.101
			0.02	0.791	1.652	0.02	0.029	0.165
0.4	0.02	0	$(1-\alpha)\lambda$	0.370	0.230	0	0	0.05
			0.02	0.379	0.337	0.014	0	0.072
		0.02	$(1-\alpha)\lambda$	0.375	0.682	0	0.029	0.147
			0.02	0.379	0.789	0.014	0.029	0.170
	0	0	$(1-\alpha)\lambda$	0.4	0.238	0	0	0.05
			0.02	0.4	0.391	0.02	0	0.082
		0.02	$(1-\alpha)\lambda$	0.386	0.690	0	0.029	0.147
			0.02	0.388	0.842	0.02	0.029	0.179

a. See table 3.

24. This outcome is fairly robust and also applies for other parameter values than those reported here.

However, comparing the first and the second half of the tables, it can be seen that changes on the demand side can lead to tremendous labor relocations. The shift observed in reality can almost be replicated by a decrease in γ from 0.8 to 0.4.[25] The explanation would be a change in preferences from a relatively large emphasis on food to a large emphasis on widgets, perhaps because new sorts of widgets have become available. These could be anything which had not been available before for mass consumption, like pottery and glassware at the beginning of the industrial revolution (see section 2.3) or goods like radios and TV-sets or automobiles during this century. This leaves changes in γ, the weight of food consumption in utility, as the only quantitatively plausible force behind industrialization.

Table 5: Steady-States for Different Parameter Values, $\sigma = 1$

γ	λ	μ	ν	n^*	z_2^*	$(\dot{c}_A/c_A)^*$	$(\dot{c}_M/c_M)^*$	r^a
0.8	0.02	0	$(1-\alpha)\lambda$	0.779	0.479	0	0	0.05
			0.02	0.779	0.479	0.014	0	0.05
		0.02	$(1-\alpha)\lambda$	0.765	0.722	0	0.029	0.079
			0.02	0.765	0.722	0.014	0.029	0.079
	0	0	$(1-\alpha)\lambda$	0.8	0.514	0	0	0.05
			0.02	0.8	0.514	0.02	0	0.05
		0.02	$(1-\alpha)\lambda$	0.781	0.758	0	0.029	0.079
			0.02	0.781	0.758	0.02	0.029	0.079
0.4	0.02	0	$(1-\alpha)\lambda$	0.370	0.230	0	0	0.05
			0.02	0.370	0.337	0.014	0	0.05
		0.02	$(1-\alpha)\lambda$	0.352	0.355	0	0.029	0.079
			0.02	0.352	0.355	0.014	0.029	0.079
	0	0	$(1-\alpha)\lambda$	0.4	0.238	0	0	0.05
			0.02	0.4	0.238	0.02	0	0.05
		0.02	$(1-\alpha)\lambda$	0.373	0.363	0	0.029	0.079
			0.02	0.373	0.363	0.02	0.029	0.079

a. See table 3.

25. One could of course start fine-tuning this model to fit its behavior to that observed in some country. Since we do not describe the development process of a particular country, though, we do not make this attempt here.

Considering changes in preferences is rather unusual in economic analysis. The assumption of constant and given preferences has already been advocated by *Alfred Marshall* and since the 1930s "hardened increasingly into dogma" (*McPherson*, 1987, 402). This position in its purest form has been advanced by *Stigler* and *Becker* (1977) who argue that tastes are literally biologically given. However, their arguments in the utility function are rather abstract wants like self-esteem or nourishment which, interacting with prices and incomes, lead to varying tastes for specific goods. Thus, the question whether preferences are constant, can be considered as depending on the degree of abstraction.

Felix (1979) rejects the *Stigler-Becker* view of abstract preferences and argues that shifting preferences have frequently accompanied and interacted with economic development, in the British industrial revolution where large parts of the population shaped their preferences by taking the landed aristocracy as model, as well as in the 20th century where the "American way of life" shaped wants in Puerto Rico, for example. We will follow *Felix* and consider the consequences from exogenous preference changes in the subsequent section. Chapter 6 contains the case of endogenous changes in consumption patterns due to increased income where the utility function remains unchanged.

3.2.2 Transitional Dynamics

In this section we explicitly solve the system of differential equations (3.6) - (3.9) numerically to analyze the economy's transition towards its steady-state. These numerical simulations allow us first of all to consider the potentially important transitional dynamics which are lost in comparative-static analyses of the steady-states. And secondly they also yield some insights about the duration of this transition period. Within this section, several questions are discussed.

The first question concerns the duration of the transition dynamics. If one accepts the possibility that preference changes might stand behind economic development, as the previous section has suggested, then a preference shock in the model should be able to replicate not only the outcome of the development process, the shift of labor into industry, but also its duration. According to *Kuznets* (1966) the transition from a largely agricultural economy with around 70 percent of the labor force in agriculture to an industrial economy with less than 20 percent has taken roughly 100 years in the US as well as in Japan (our second stylized fact above).

A precondition for this outcome is that the transitional dynamics can really last for such a long time. After all, they are usually neglected and it is assumed that countries are on their steady-state growth paths all the time. There is some evidence from numerical experiments with the neoclassical one-sector growth model which indicates that this transition can indeed last for a long time. Early work by *Sato* (1963) and *Atkinson* (1969) pointed to transition periods of the order observed by *Kuznets*. *King* and *Rebelo* (1993) replicate the *Sato* experiment within a slightly different set-up and study the transitional dynamics in a simple model that exhibits a sevenfold per-capita output rise within 100 years, which roughly corresponds to the development of per-capita income in the United States from 1870 - 1970. They choose the parameter values such that transitional dynamics and technical progress each account for half of the increase. Their simulation shows that the transitional dynamics are important and present for a long time: they raise the growth rate of output on average by 3.2 percentage points over the first ten years, and then 1.7, 1.1, and 0.8 percentage points over the following three decades. *King* and *Rebelo* also note that the length of this period depends very much on the specification of the saving mechanism and the rate of intertemporal substitution in individuals' preferences. In a model with endogenous saving and a high rate of substitution the transition period can become much shorter.

The next question to be answered by the numerical simulation is whether the model shows at all a realistic transitional behavior. It is possible that a model's short term empirical implications are considerably different from the implications taken from its steady-state behavior. An example for unrealistic transitional dynamics is again the study by *King* and *Rebelo* (1993), who found that the neoclassical growth model even with a realistic parameterization can imply interest rates around 800 percent in the first years of the transitional period.

The final question concerns the quantitative effects of economic policies on the development process. We know so far that economic policies have only very small effects on the steady-state outcome. However, it may well be that they influence the duration of this transition and thus, for example, speed up the process of structural change. Again this cannot be inferred from steady-state analysis. Since the methodology to solve such a system of equations is slightly more complicate than numerical discussions of the steady-state, we first discuss the mathematical nature of the simulation problem and then present a procedure to solve it. The simulation results and their discussion follow.

This discussion is based on the belief that a theoretical model's implication should not contradict empirically observed facts. Therefore accordance with stylized facts is considered necessary for a "good" model. However, this belief needs some qualifications: First of all, accordance with stylized facts is not sufficient. Otherwise an absolutely foolish model who's predictions just happen to coincide with some facts from reality must be considered a "good" model. And secondly an economy is frequently exposed to external shocks not modeled here, especially over a longer time as modeled here. Therefore a failure of reconciling model predictions and reality might be due to these shocks, while the model mechanism is correct. These qualifications should be bourn in mind for the subsequent discussion.

In the two-sector model of section 3.1, equations (3.6) - (3.9) characterize the dynamic behavior of the economy. By differentiating (3.6) and (3.7) with respect to time and applying some simple substitutions these four equations reduce to a system of ordinary differential equations (ODEs) in the observable variables c_M, K, and n that fully describe how the economy evolves:

$$\begin{aligned} \dot{n} &= f_1\,(c_M, n, K, A, M) \\ \dot{K} &= f_2\,(c_M, n, K, A, M) \qquad (3.26) \\ \dot{c}_M &= f_3\,(c_M, n, K, A, M) \end{aligned}$$

The growth of A and M is exogenous as before. This system of equations implies optimal functions $c_M(t)$, $K(t)$, and $n(t)$ which, together with the initial value for capital $K(0)$ and the transversality condition, are the solution to the optimal control problem (3.4).

Depending on the nature of the boundary conditions, problems with ODEs can be divided into two groups: initial value problems and two-point boundary value problems.[26] In initial value problems the starting values for all variables are given

26. For an overview of these problems and numerical solution techniques see *Goffe* (1993), *Press et al.* (1992), or *Dixon et al.* (1992).

for the same point of time. To solve the problem, they simply have to be inserted into the equations before the system is integrated forward numerically using standard techniques like the Runge-Kutta algorithm (cf. *Press et al.*, 1992). For the ODE system (3.26) this solution technique would imply choosing values for $c_M(0)$, $K(0)$, and $n(0)$ and then integrate forward. However, we know that this will lead the economy in most cases far away from the steady-state. While $K(0)$ can be freely chosen, $c_M(0)$ and $n(0)$ cannot. They are control variables that have to take on specific values to keep the economy on the equilibrium path, the stable trajectory towards the steady-state.

Therefore (3.26) is of the second kind: it is a two-point boundary value problem. One condition – $K(0)$ – applies at time zero, another – the transversality condition – at infinity. This makes the search for the solution path even more difficult since we cannot solve the model up to infinity and then check if the second boundary condition is met, as one would do with a boundary condition within a finite time period. However, since the steady-state satisfies the transversality condition, as discussed in the previous section, it can be used as the second boundary value.

There are several algorithms to solve this problem numerically, ranging from simple shooting (cf. *Press et al.*, 1992) to more sophisticated methods like the extended-path algorithm (*Fair* and *Taylor*, 1983; *McKibbin*, 1992), projection methods (*Judd*, 1992), perturbation methods (*Judd* and *Guu*, 1993), or the time-elimination method (*Mulligan* and *Sala-i-Martin*, 1991, 1993). For numerical computations in this and the following chapters we will use the time-elimination method since it is the simplest to compute.

The time-elimination method is a four-step algorithm which makes use of transformations of the dynamic system already applied for the stability discussion. It transforms the two-point boundary value problem into an initial value problem that can be solved subsequently with standard methods. The first step is to transform system (3.26), which has a constant growth path solution, into one where all variables are stationary in the steady-state. This is exactly what we have done for the stability analysis above by creating the new control-like variable z_1 and the new state-like variable z_2. Equation (3.18) shows that at every moment in time z_1

is determined by z_2 and n. The differential equations for z_2 and n are given by equations (3.19) and (3.20) and can be written in short as:

$$\begin{aligned} \dot{n}(t) &= \xi_1(n(t), z_2(t)) \\ \dot{z}_2(t) &= \xi_2(n(t), z_2(t)) \end{aligned} \tag{3.27}$$

The second step is to argue that the steady-state solution (z_2*, n*) of (3.27) is an optimal solution to the control problem for some feasible initial conditions. This has been shown above. It is also known that the steady-state is stable if some parameter restrictions are met. Then there exists a stable manifold of the steady-state which – by definition – is the locus of points in the [z_2, n] space which approach the stationary point if they evolve according to (3.27).

Next, the system (3.27) is transformed into a policy function. This policy function is the central element in the time-elimination method. It expresses the system of ODEs as a policy problem for a social planner: the function gives the value of the control variable a social planner has to choose for each possible value of the state variable:

$$n(t) = n(z_2(t)) \tag{3.28}$$

While this policy function cannot be derived algebraically, its slope can be obtained from (3.27) and (3.28) by applying the chain rule of calculus:

$$n'(z_2) = \frac{\dot{n}}{\dot{z}_2} = \frac{\xi_1(n, z_2)}{\xi_2(n, z_2)} \equiv \xi(n, z_2) \tag{3.29}$$

Thus, equation (3.29) eliminates time and yields an ordinary differential equation for n depending on z_2. We also know that $n^* = n(z_2^*)$ in the steady-state which can be used as initial condition for the problem. However, an additional modification is necessary to obtain an initial value problem: equation (3.29) is not defined for the initial steady-state since in this situation $\dot{n} = \dot{z}_2 = 0$ by definition. The required modification is to specify the slopes of the policy functions at the steady-state.

We obtain this slope from the stable eigenvectors of (3.27). Calculating the Jacobian of the dynamic system (3.27) and evaluating it at the steady-state (which we already did for the stability analysis) yields a matrix which is equivalent to the linearized system of (3.27). Eigenvalues and eigenvectors can be computed easily from the Jacobian. Since the model is saddle-point stable there exists one positive eigenvalue corresponding to the unstable manifold as well as one negative eigenvalue for the stable manifold. The eigenvector belonging to this negative eigenvalue is tangent to the stable manifold of the system and can be used to compute its slope at the steady-state.[27]

Finally, the fourth step of this method is to solve the augmented initial value problem with standard numerical routines. We have used the routine NDSolve in *Mathematica.* Time paths for n or z_2 as well as for any other variable in the system can then be computed by specifying an initial value for the state-like variable z_2 and finally integrating the model forward in time using the control values prescribed by the policy function $n(z_2)$.

Now consider the actual transition process. The policy function (3.29) for our problem can be obtained from equations (3.19) and (3.20) as

$$n'(z_2) = \frac{n(1-n)\,[\sigma\lambda(1-\alpha)-\mu-\rho+(1-\sigma)(\mu(1-\gamma)+\gamma\nu)]}{(\sigma(1-\alpha n)+(1-\sigma)\gamma(1-\alpha))\,z_2\left(\mu+\alpha\lambda-\frac{\alpha z_2(1-n)^{\alpha}}{(1-n)}\left(\frac{\gamma-n}{\gamma}\right)\right)} + \frac{(1-\alpha)(1-\gamma)\,n(1-n)^{\alpha}\frac{(\sigma n+\gamma(1-\sigma))}{\gamma}}{(\sigma(1-\alpha n)+(1-\sigma)\gamma(1-\alpha))\left(\mu+\alpha\lambda-\frac{\alpha z_2(1-n)^{\alpha}}{(1-n)}\left(\frac{\gamma-n}{\gamma}\right)\right)}. \quad (3.30)$$

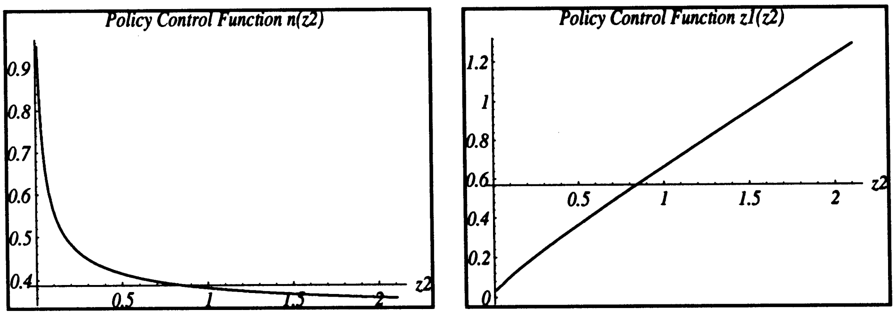
Solving the differential equation (3.30) as described above leads to a numerical approximation of the policy function $n(z_2)$. Making use of equation (3.18) also leads to the corresponding policy function for the control-like variable $z_1(z_2)$. These functions are depicted in figure 4 for the parameter values $\sigma = 5$, $\gamma = 0.4$, $\lambda = 0$, $\mu = \nu = 0.02$.[28]

Figure 4 shows that if $z_2 = ML^{\alpha} / K^{\alpha}$ is initially below its steady-state value, for example due to some shock, the social planer has to choose a higher value for n. Over time the economy moves down the curve until it reaches its steady-state at the intersection with the horizontal axis. By applying some simple transforma-

27. Suppose that the stable eigenvector of the system (3.27) is (3, -2). Then $n'(z_2)$ evaluated at the steady-state is -3/2.

28. We choose $\lambda = 0$ for better comparability to the models in the next chapters. The dynamics are not fundamentally different for a strictly positive λ.

Figure 4: Control Functions



tions to the equations (3.6) - (3.9) from the maximization problem, we can also obtain functions for the growth rates of c_A, c_M, and K as well as for the interest rate (the marginal product of capital). Movements along these curves describe the corresponding transitional behavior for growth and interest rates. They are plotted in figure 5.

Figure 5: Transitional Behavior of Growth and Interest Rates

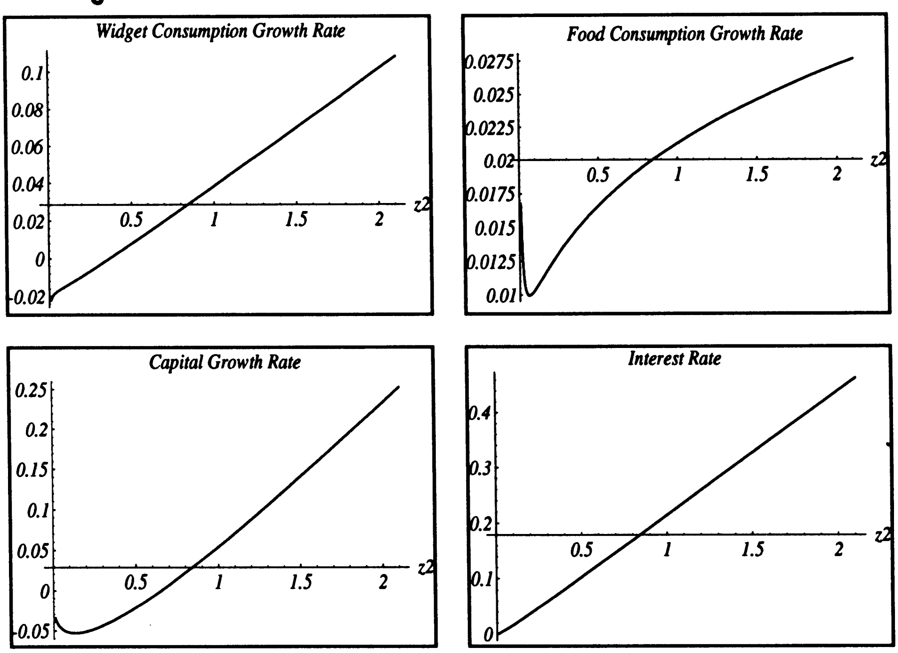


Figure 5 shows that most of these rates are in a realistic range if the state-like variable z_2 is not too far away from its steady-state value. An exception is the interest rate which has already an unrealistically high steady-state level. At least it does not rise to 800% like it did in the *King* and *Rebelo* analysis.

A disequilibrium state where $z_2 = ML^{\alpha} / K^{\alpha}$ is less than its steady-state value must be interpreted as a situation where the capital stock per capita is initially too large for the existing level of technology or, in another interpretation, a large capital stock is used with a low total factor productivity (Note that "large" is meant relative to the corresponding *M*).[29] The other case would be a value for z_2 which is initially too large. Along the same lines this should be interpreted as a too small capital stock. For both situations we can calculate durations for the transitional dynamics. These calculations are conducted for the same parameter values as in tables 3 - 5. We assume that in the beginning z_2 is at $z_2^* \pm 90\%$ and we record the number of years it takes the economy to reach the values $z_2 = z_2^* \pm 20\%$ and $z_2 = z_2^* \pm 10\%$. The results are given in tables 6 - 8.

Table 6: Duration of Transitional Dynamics, $\sigma = 10$

γ	λ	μ	ν	$z_2^* - 20\%$	$z_2^* - 10\%$	$z_2^* + 20\%$	$z_2^* + 10\%$
0.8	0.02	0	$(1-\alpha)\lambda$	125.58	149.89	28.50	45.74
			0.02	46.28	54.56	8.99	14.66
		0.02	$(1-\alpha)\lambda$	49.09	58.33	10.34	7.69
			0.02	29.62	34.86	5.44	8.93
	0	0	$(1-\alpha)\lambda$	144.47	170.50	29.65	47.80
			0.02	37.64	44.15	6.82	11.19
		0.02	$(1-\alpha)\lambda$	52.01	61.52	10.53	17.07
			0.02	25.95	30.41	4.44	7.36
0.4	0.02	0	$(1-\alpha)\lambda$	218.55	265.75	59.02	93.73
			0.02	127.99	153.69	30.91	49.41
		0.02	$(1-\alpha)\lambda$	55.04	65.91	12.70	20.41
			0.02	46.59	55.62	10.37	16.73
	0	0	$(1-\alpha)\lambda$	293.86	349.93	66.66	106.69
			0.02	120.73	143.54	26.71	42.88
		0.02	$(1-\alpha)\lambda$	58.58	69.86	13.03	20.99
			0.02	45.76	54.41	9.79	15.82

29. It is not really clear what "large relative to *M*" means, a problem one could call "*Kaldor's* Revenge". *Kaldor* (1961, 205) strictly opposed the neoclassical production function with technical progress as a factor, since "... unlike labor, the state of knowledge is not a quantifiable factor. A *given* or *constant* state of knowledge is only capable of being defined implicitly." We shall therefore identify a low z_2 with a large capital stock and a high z_2 with a small capital stock.

Several observations can be made from these calculations: First of all, the duration of the transition dynamics is asymmetric. It takes an economy about 4 - 6 times as long to reach its steady-state from below than from above. Secondly, the calculations show that the transitional dynamics can indeed last for a rather long time, which confirms the *Sato* experiment and related studies. This leads to the conclusion that development could be understood as a transitional phenomenon between two steady-states. However, a final judgement, whether such a transitional phenomenon can replicate the extend as well as the duration of industrialization as described by the first two stylized facts, requires a realistic experiment which we conduct below.

Table 7: Duration of Transitional Dynamics, $\sigma = 5$

γ	λ	μ	ν	$z_2^* - 20\%$	$z_2^* - 10\%$	$z_2^* + 20\%$	$z_2^* + 10\%$
0.8	0.02	0	$(1-\alpha)\lambda$	89.97	106.63	18.89	30.51
			0.02	49,78	58.59	9.53	15.53
		0.02	$(1-\alpha)\lambda$	43.75	51.76	8.73	14.2
			0.02	31.60	37.15	5.74	9.45
	0	0	$(1-\alpha)\lambda$	97.92	115.15	19.14	31.00
			0.02	43.27	50.67	7.77	12.74
		0.02	$(1-\alpha)\lambda$	45.85	54.01	8.81	14.36
			0.02	28.98	33.93	4.94	8.21
0.4	0.02	0	$(1-\alpha)\lambda$	147.67	177.27	35.42	56.66
			0.02	108.69	129.86	24.86	39.89
		0.02	$(1-\alpha)\lambda$	52.30	62.41	11.55	18.63
			0.02	46.44	55.30	9.97	16.14
	0	0	$(1-\alpha)\lambda$	177.06	210.19	38.50	61.86
			0.02	108.16	128.26	23.07	37.16
		0.02	$(1-\alpha)\lambda$	55.81	66.37	11.91	19.25
			0.02	46.64	55.32	9.67	15.67

Thirdly, we can obtain some information about the influence of parameter changes on the transition duration. An increase in σ, which is a decrease in the intertemporal elasticity of substitution, increases the transition duration for a low rate of technical progress in agriculture and decreases it for a high rate. This is an interesting result since it is contradictory to the outcome for a one-sector model as derived, for example, by *King* and *Rebelo* (1993). There an increase in σ generally lengthens the duration of the transitional period. The intuition behind our result is the following: A low intertemporal elasticity of substitution means that people are unwilling to defer consumption in favor of capital accumulation. If the rate of technical progress in agriculture is high, however, consumption of widgets can be substituted by consumption of food, which alleviates the problems of intertemporal substitution. Therefore an increase in σ does not lead to longer transition dynamics in this case.

We can also observe that increasing the rates of technical progress in either sector drastically shortens the transition period. However, it is not clear which increase has stronger effects. The effect of a decrease in λ depends again on ν. For a low rate of progress in agriculture a lower λ implies longer and for a high rate of technical progress shorter transitional dynamics.

Table 8: Duration of Transitional Dynamics, $\sigma = 1$

γ	λ	μ	ν	$z_2^* - 20\%$	$z_2^* - 10\%$	$z_2^* + 20\%$	$z_2^* + 10\%$
0.8	0.02	0	$(1-\alpha)\lambda$	52.70	61.56	9.36	15.31
			0.02	52.70	61.56	9.36	15.31
		0.02	$(1-\alpha)\lambda$	32.88	38.42	5.59	9.23
			0.02	32.88	38.42	5.59	9.23
	0	0	$(1-\alpha)\lambda$	55.01	63.87	9.18	15.06
			0.02	55.01	63.87	9.18	15.06
		0.02	$(1-\alpha)\lambda$	33.89	39.44	5.52	9.13
			0.02	33.89	39.44	5.52	9.13
0.4	0.02	0	$(1-\alpha)\lambda$	61.28	72.94	13.21	21.35
			0.02	61.28	72.94	13.21	21.35
		0.02	$(1-\alpha)\lambda$	38.30	45.42	7.92	12.89
			0.02	38.30	45.42	7.92	12.89
	0	0	$(1-\alpha)\lambda$	63.56	75.45	13.38	21.63
			0.02	63.56	75.45	13.38	21.63
		0.02	$(1-\alpha)\lambda$	39.32	46.65	8.00	13.04
			0.02	39.32	46.65	8.00	13.04

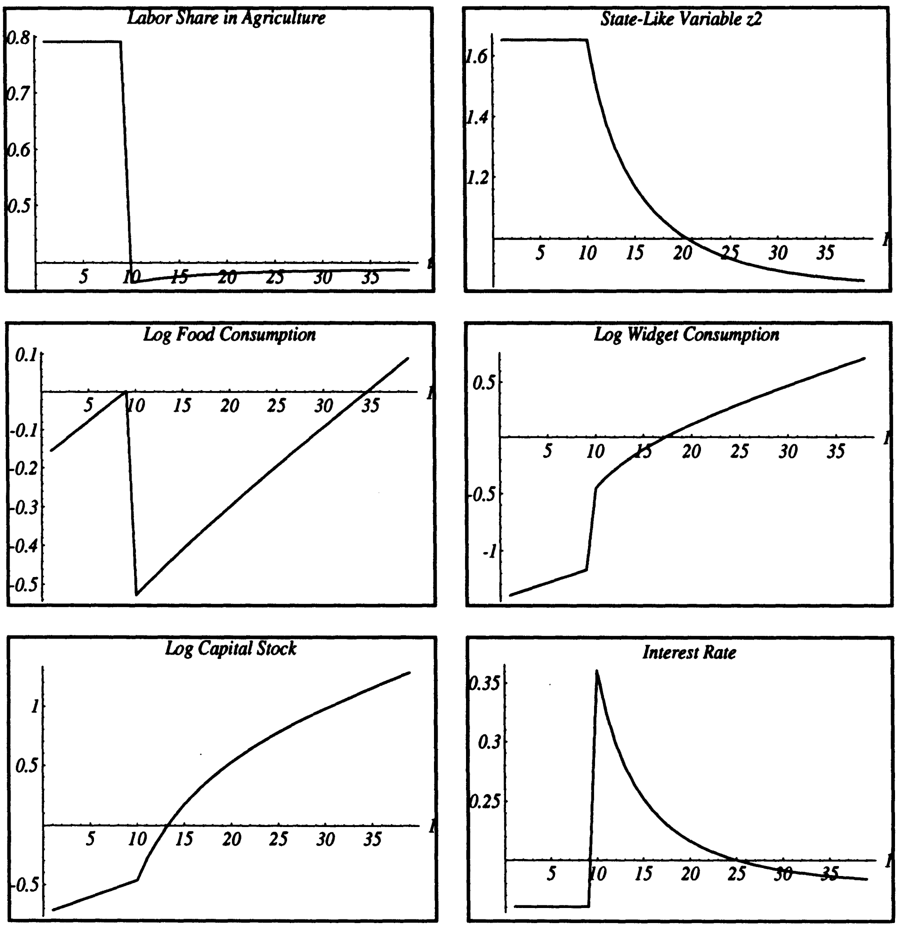
These numerical results are interesting since they relativize the model's algebraic results: In section 3.2.1 we have shown that industry led development, that is, an increase in μ, leads to a larger fraction of labor in industry in the steady-state. Rising ν, however, the rate of technical progress in agriculture, has an ambiguous effect which for $\sigma > 1$ even leads to a less industrialized economy. The numerical calculations have shown that the advantages of increases in μ might be smaller than the algebraic results suggest: the numerical effects of such a policy on n^* are rather small. Thus, if there is a tendency in the economy to industrialize, for example due to a shift in preferences, it would be more important to speed up the transitional process. Here the advantage of an increase in industrial technical progress is less clear. At least for a large σ and γ an increase in ν reduces the transitional period by more than an increase in μ.

3.2.3 Economic Development as Transitional Process

So far we have shown that the transitional dynamics are long-lasting and can be influenced by economic policy. This leads to the possibility of modelling economic development as a transitional process between two steady-states. This, however, requires some ultimate cause behind the transition, be it a shock in the levels of state-variables, in production parameters, or in preferences. The case usually studied is the first one. *King* and *Rebelo* (1993) or *Mulligan* and *Sala-i-Martin* (1993), for example, discuss the situation of an economy that had been at its steady-state until some event (e.g., a war) moved it away from the stationary equilibrium. However, in the model considered here, this would imply that a now underdeveloped economy has the same steady-state structure as an industrial country. It is just kept away from this equilibrium by some forces – by now for a couple of hundred years.

Within the model considered here, the only reasonable shocks are changes in parameters which lead the country to a new steady-state. We have already seen that one possible external factor behind development, changes in the production parameters μ, ν, and λ cannot cause sufficiently large changes in the steady-state fraction of labor.[30]

Figure 6: Preference Shock with Fast Technical Progress

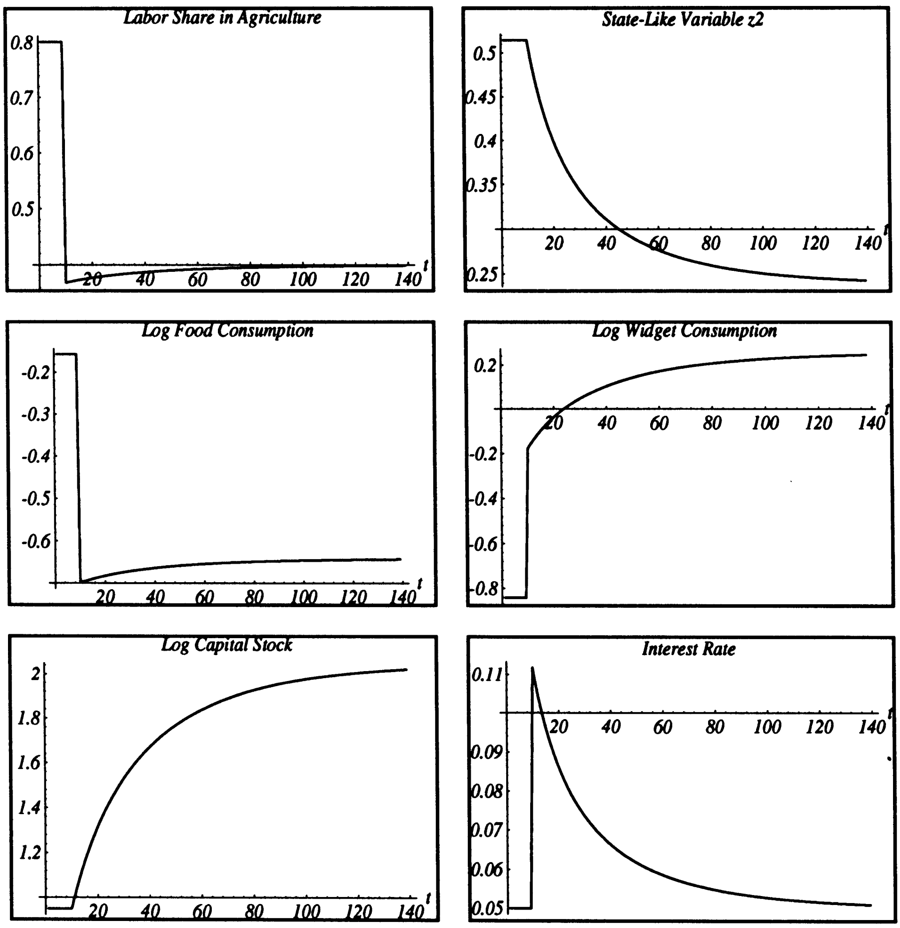


This leaves as explanation a change in preferences, for example from $\gamma = 0.8$ to $\gamma = 0.4$. As we have seen above, such a change almost replicates the observed shift of labor from agriculture to industry from the first stylized fact. Then the way to discuss the results of the shock is to study the transitional behavior of the after-shock economy in which the shock is the steady-state level of the state variable z_2 from the pre-shock economy. Comparison of the respective lines in table 4 shows that this situation corresponds to a z_2 which is initially above its steady-state. We conduct two simulations, one without technical progress ($\mu = \nu = 0$) and one with

30. However, we show in chapter 6 a situation where changes in ν do have effects, namely when the income elasticity of food demand is less than unity (Engel's law).

moderately high rates ($\mu = \nu = 0.02$). In both cases we have $\sigma = 5$ and $\lambda = 0$. This corresponds to the pre-shock situations given by lines 5 and 8 in table 4. The development paths are depicted in figures 6 and 7. These graphs show the development of several variables over time, where the first ten periods characterize the pre-shock steady-state. The levels of *A*, *M*, and *L* have been normalized to one for the first period.

Figure 7: Preference Shock without Technical Progress



Several observations can be made. First of all, the graphs show again that transition occurs rather rapidly in this model. According to figure 6 the new steady-states are almost reached within 20 years after the shock has occurred. Without technical progress this transition take longer, about 100 years as figure 7 shows.

This is approximately the duration stated in the second stylized fact. However, figure 7 shows that these 100 years apply to capital stock and interest rate and not so much to the labor share in agriculture, n. This variable adjusts very quickly. The experiment from figure 7 also violates the first stylized fact, continuous increases in labor productivity. Replicating this fact, we are back to figure 6 with even faster transitional dynamics. We could now start to calibrate the model to these facts by choosing different parameter values. While technically feasible, this does not seem to be very promising since it does not change the behavior of n.

This rather unrealistic behavior is due to n being a control variable. In reality labor cannot be shifted so quickly from one sector to another, not even by a social planner.[31] We can also observe a kind of overshooting. Initially, the fraction of labor in agriculture is at its high pre-shock steady-state. Then, due to the shock n decreases considerably and subsequently approaches its new low steady-state value from below. The economic intuition behind this "overshooting" is that since the preference change requires more widgets being produced and also more capital to do this, labor shifts to industry to pursue these goals. After a time, when a sufficiently large capital stock has been build up, part of the labor force goes back to the agricultural sector. The extend of this overshooting decreases as the preferences change more gradually.

Comparing the development of food and widget consumption (which are printed in logarithmic scale) we see that food consumption falls drastically and immediately when labor is shifted into industry. This is the effect discussed in section 3.2.1. The immediate decrease is caused by the quick labor shift. It takes the economy about 30 years in figure 6 to reach the pre-shock levels of food consumption per capita. Since there is no technical progress in figure 7 it never returns to its pre-shock level in this case. The opposite happens to widget consumption where a positive level effect exists.

To conclude, the results about this model's realism are mixed. While changes in preferences have been singled out as the only possible force behind the observed

31. One should keep in mind that this model is a social planning exercise describing not necessarily realistic time-paths for the variables. There are, though, indeed examples in planned economies where dramatic labor shifts occurred, for example, in China, the Soviet-Union, and Cambodia.

structural change, they must happen gradually if the model is to show any realistic behavior. But this is not an unrealistic requirement. In reality such a shift of preferences, especially of the size assumed for the simulation experiment, indeed takes a much longer time implying a longer transitional period.

3.3 Summary

In this chapter a simple model of a dual economy with exogenous technical progress in agriculture and industry has been presented. The steady-state equilibrium derived from this set-up was shown to be saddle-path stable for a wide range of parameter values. It turned out that the growth rate of consumption for either good was independent of parameters describing production of the other good. The reason for this behavior is that in this chapter only production of both goods is asymmetric, not consumption.

Analytical and numerical analysis of the model's behavior showed that increases in the rate of technical progress in industry make a country more industrialized in the steady-state. An increase in the rate of labor force growth has the same effect whereas a rise of the rate of technical progress in agriculture leads to a larger fraction of labor in this sector. While the first effect is in accordance with the implications of standard dual economy models, the last two are not. In both cases this is caused by the absence of Engel effects which are present in the classical dual economy models. We will therefore incorporate them into the model in chapter 6.

It was found that the quantitative effects of these economic policies on the degree of industrialization are rather small. However, increases in the rate of technical progress lead to a considerably shorter transition period suggesting that the main merits of such policies are to be found here.

The single force strong enough to generate a shift of labor from agriculture to industry as strong as observed in reality turned out to be a change in preferences. Simulation of such a "preference shock" showed that a single, large shock generates an unrealistic behavior implying that there must occur several shocks of small magnitude to replicate also the time-scale of economic development observed in reality. Changes in preferences, so the main outcome of this model, are a precondition for industrialization. On the one hand this seems to be a reasonable outcome since it implies that industrialized countries are industrialized because there exists a demand for the output of this sector, for example, for cars. Technical progress in industry alone would not be sufficient for industrialization.

On the other hand this outcome should not be interpreted as meaning that underdeveloped countries are under-industrialized in accordance with their preferences and that a preference change alone is sufficient to industrialize. In these countries a change in preferences has probably occurred already and one should therefore look at factors influencing the length of the transition period like the rates of technical progress.

In any case the outcome of this chapter suggests that it might be useful to distinguish between the determinants of the final, steady-state structure of an economy (preferences) and the forces influencing the transition between such steady-states (here, e.g., the rates of technical progress). This distinction is not made by the classical dual economy literature.

A comparison of the model simulation with the stylized facts mentioned in the introduction to this chapter shows that the first and third facts – a shift of labor into industry and increases in labor productivity in both sectors – are met relatively well by the preference change scenario assuming that both rates of technical progress are positive. Accordance with the third fact is therefore more an assumption than a result of the model. The second stylized fact – the duration of this industrialization process – can only be replicated if preferences change gradually. Since such a gradual change is more realistic than a sudden large shift, the model can be regarded as being in accordance with the second stylized fact, too.

4. Endogenous Growth in the Dual Economy

In this section the baseline model is extended to endogenous technical progress in agriculture. Technical progress in manufacturing remains exogenous. While it would, in principle, be possible to introduce some research or human capital accumulation decision also in the second sector, this study focuses on agriculture as set out in chapter 2. Extending the model towards endogenous development allows us to discuss determinants of productivity improvements in agriculture as well as their macroeconomic effects. Within the set-up chosen here, this discussion is not confined to growth effects but includes also the economy's structure. The focus is in this chapter on technology creation as well as on human capital investment. Both issues have been discussed intensively in empirical and theoretical studies alike, although mostly from a microeconomic perspective. Most macroeconomic studies have taken productivity improvements in agriculture as exogenous and have considered only their consequences. Here both issues are discussed together. Using tools from the NGT, determinants of productivity improvements as well as their effects will be analyzed within a single macroeconomic model. As in the last chapter, the model allows us to distinguish between growth and level effects as well as between influences on an economy's structure and on its growth performance. Like above, we will confront the model with the stylized facts set out at the beginning of chapter 3.

In this chapter we proceed as follows: In the first section we discuss different possibilities to increase productivity in the agricultural sector, namely research on and development of new, more productive techniques, as well as investment in human capital. We derive a formal specification encompassing both techniques to a certain extend. In section 2 this new force driving technical progress is added to last chapter's model. We show that in the presence of externalities two solutions exist of which one is optimal and the other, which is the market outcome, is sub-optimal. Section 3 discusses the consequences of certain economic policies in this model, first for the long-run steady-state and then for the transition dynamics as before analytically as well as numerically. Finally, section 4 summarizes.

4.1 Endogenous Technical Progress in Agriculture

If the improvement of agricultural productivity is to be endogenized, one first has to discuss the productivity improving process. What exactly are the endogenous decisions that lead to a rise in productivity? One possibility would be learning by doing. However, learning by doing alone in the context of agriculture just does not seem to be very plausible. Most countries have been agricultural economies for a fairly long time in their history without experiencing considerable increases in productivity. One possible explanation for this stagnation is that learning from a a given set of techniques is bounded. As *Young* (1991) notes, there is considerable evidence that the learning curve, although approximately log-linear after the introduction of new techniques, ultimately reaches a phase where additional experience yields absolutely no gains in productivity. Therefore learning only keeps going with continuous introduction of new goods (*Young*, 1991; *Lucas*, 1993).[1]

Another possibility to increase productivity is improvement of human capital. As already mentioned in chapter 2, human capital could increase output by several different mechanisms. First of all it could have a direct effect on the level of output. More human capital raises the efficiency of using land, labor, and technology. The simple idea behind this mechanism is that someone who is able to read can, for example, use fertilizer more efficiently since she is able to read the usage instructions. Secondly, human capital could influence the growth rate of technology or productivity. Having an education, for example as agricultural engineer, facilitates continuous improvement of irrigation and planting techniques. And last, human capital could influence ability and willingness to adopt new techniques, be it from technological leaders (to catch up in productivity) or simply from government-financed agricultural research institutions. While the first two mechanisms are discussed here, the third one is left to chapter 5.

A third way to increase productivity is technology creation by research which is exactly the counterpart to human capital accumulation; the latter takes the availability of new production techniques as given and the former the ability to adopt these technologies. New agricultural technologies have been very important in the last years, both in developed and developing countries as discussed in chapter 2.

1. This observation also shaped *Arrow's* (1962) learning by doing model. Cf. section 2.1.

In human capital accumulation as well as in research, externalities can arise. The nature of these externalities is slightly different, though. Concerning human capital, *Lucas* (1988) has argued that in addition to a person's level of human capital the average level of human capital in the economy should enter the production function as an externality. It is an externality since a single individual would not take it into account when making decisions about human capital accumulation. Within agricultural production a higher average level of human capital would facilitate introduction of new techniques: If, for example, a village adopts a production technique with a relatively extensive division of labor, coordination of these activities is much simpler if everybody understands the whole project and therefore her own role in it than in a situation where only one human capital rich coordinator exists and the rest does not really comprehend the new project. *Jimenez* (1994) points out more general externalities: primary education may foster "good citizenship", that is, increase patriotism, decrease crime, or, through literacy, ease the administrative burden of tax collection. Of course a social planner would take such externalities into account and decide differently about human capital accumulation.

But also investing in agricultural research and thus increasing technological knowledge for agricultural production is an activity which involves considerable externalities. Agricultural knowledge mostly has a "design" character, as *Romer* (1990) called it: although developed knowledge is a factor of production, it differs in a crucial way from other factors: its usage in production is nonrival which means that several producers can use the same "factor" at the very same time. This is how externalities to research can arise. A new high-yielding seed variety, for example, has to be developed only once and can then be produced with roughly the same cost as traditional seeds by every farmer. The same applies to new fertilizers or a different kind of crop rotation. While the private rate of return to this activity is very low – it might even be negative – the social rate is usually much higher as the evidence mentioned in section 2.4 shows.

The model derived in this chapter can be understood as representing both features, investment in research and development (R&D) with its externalities as well as human capital accumulation with externalities. The main point is that labor resources used in production could also be used to increase productivity and therefore output in the future. This activity bears some resemblance to saving where also resources which could be consumed are instead invested to increase future production and consumption. The basic equation is again the agricultural production function (3.1) from last chapter. If we assume for simplicity that the labor force is constant and normalize it to unity,[2] this function becomes

$$(4.1) \quad Y_A = A^{\eta}(un)^{\alpha}, \qquad 0 < \alpha < 1, \eta > 0.$$

Again n is the fraction of labor employed in the agricultural sector. It also denotes total labor in agricultural production. Of this number only a fraction u is engaged in the actual production process. A captures the state of agricultural technology or, alternatively, the level of human capital. η, its output elasticity, can be divided into two parts according to $\eta_1 + \eta_2 = \eta$ $(\eta_1 \geq 0, \eta_2 > 0)$. Here η_1 denotes the externality part of the output elasticity and η_2 the part that is taken into account by individuals. The state of technology, A, can be increased by engaging in R&D or in human capital accumulation which is done by the fraction $(1 - u)$ of the agricultural population. The level of A therefore does not grow anymore with the constant exogenous rate ν but according to the following equation:

$$(4.2) \quad \dot{A} = A\delta(1-u), \qquad \delta > 0$$

where δ is a parameter describing the efficiency of this process. The output of new knowledge or human capital is hence a function of the effort devoted to research or to human capital accumulation.[3] The formalization in equation (4.2) is the same as *Lucas'* (1988) function for human capital accumulation.[4] Linearity in A is

2. We will stick to this assumption for the remainder of the study since it simplifies the algebra considerably. It is also justified by the topic of this study which is technical progress, not labor force growth. Accounting properly for the latter would also require discussion of the fertility determinants which is beyond the scope of the analysis conducted here.
3. Note that equation (4.2) states that the (average) engagement *per capita* in research and development or human capital accumulation determines the productivity growth rate, not the absolute number of hours, $n(1 - u)$. This assumption seems to be reasonable especially for human capital accumulation. The speed and extend of technology adoption depends on the level of human capital of *each* farmer and not on the size of the agricultural sector. Otherwise a large agriculture where a small fraction engages in research or human capital accumulation would grow with the same rate as a small one with a large fraction.
4. It is therefore subject to the same critique given, for example, by *Solow* (1991). He criticized that the function has to be exactly homogeneous of degree two to generate perpetual growth which he considered as being not very realistic.

what makes unbounded growth possible. Hence, unbounded growth is more of an assumption than a result of the model.

While the next two sections will be based on (4.1) and (4.2), the exact interpretation of these equations differs depending on the kind of productivity improving process discussed. Consider investments in R&D first: Suppose that technical progress is land-saving by increasing the productivity of each available acre of farm-land.[5] Then, since the agricultural production function was assumed homogeneous in land and labor, it can be written in the following way along the lines of *Romer* (1986, 1990) and *Griliches* (1979):

$$Y_A = A^{\eta_1} A^{1-\alpha} (un)^{\alpha}, \qquad (1-\alpha) + \eta_1 = \eta$$

where $1-\alpha = \eta_2$ denotes the output-elasticity of the directly land-increasing effect. For the decision to invest in R&D only this effect is taken into account. In addition there exists an externality denoted by A^{η_1} which captures the above discussed non-rivalries. Again, a social planner knows about the externalities and therefore uses equation (4.1) as the relevant production function.

Consider next human capital in agricultural production. Denoting the level of human capital by h and the average level of human capital in the economy by $\bar{h}$, equation (4.1) changes into

$$Y_A = \bar{h}^{\eta_1} (unh)^{\alpha}, \qquad \alpha + \eta_1 = \eta$$

and equation (4.2) into

$$\dot{h} = h\delta(1-u)$$

where human capital accumulation can be interpreted as going to school or obtaining higher education.

5. Note that much of the literature in development economics on technology improvements in agriculture discusses this point (for a short introduction see *Rayner* and *Ingersent*, 1991, 31ff.). It is mostly accepted that technical progress in agriculture should be land-saving and labor-using since most of subsistence agriculture is characterized by extremely low yields, scarcity of land, and, temporarily, abundance of labor without alternative occupation possibilities.
The empirical evidence, though, about changes in employed factor proportions due to new technologies is mixed, even for technologies like tractors which one would classify as labor saving at first sight. Also, many productivity improving projects in developing countries tend to be labor-saving in the long-run since they often require capital as complimentary input.

The average human capital level would be regarded as exogenous by individuals while in fact each human capital accumulation decision also influences the average level of human capital in the economy. A social planner would be aware of this externality and would thus take it into account. In equilibrium identical individuals will have the same amount of human capital, so that $\bar{h} = h$. Then the two equations are identical to equations (4.1) and (4.2) with $h = \bar{h} = A$ and $\alpha = \eta_2$.

Last, consider the influence of human capital on the R&D process. One could model this process as

$$\dot{A} = A\phi_1 h$$

where the level of human capital influences the growth rate of technology as in *Romer* (1990). The parameter ϕ_1 describes the efficiency of this research process. As before, only part of the productivity effect from research is taken into account. A constant growth equilibrium would only be possible with a constant level of human capital. Once an individual has accumulated this steady state level, it could devote all its working time to production. The other possibility, an ever-growing h, would lead to an ever-accelerating growth of technology which is quite unrealistic.

However, in reality human capital is subject to depreciation, not only because new technologies often require different knowledge but also because humans' life expectancy is limited. If an "individual" in this model is interpreted as a "dynasty", that is, a sequence of generations, it would have to visit school from time to time again just to keep a certain level of human capital. Suppose that to keep a level h of human capital, the "dynasty" has to devote a constant fraction of its working time to education. Then the level of human capital is given by

$$h = \phi_2(1-u)$$

where ϕ_2 denotes the productivity of schooling and $(1-u)$ the fraction of time devoted to education to keep h constant. Both equations combined yield

$$\dot{A} = A\phi_1\phi_2(1-u)$$

which is identical to equation (4.2) with $\phi_1\phi_2 = \delta$. The main difference to the first R&D interpretation is the parameter δ which now contains the efficiencies of schooling and of research together.[6]

4.2 The Model

All the interpretations from previous section can be discussed in the framework of equations (4.1) and (4.2), and all of them show some kind of externality. Therefore the next two subsections discuss the dual economy model based on equations (4.1) and (4.2), first its optimal solution where the externality is taken into account, and then the market solution where this is not the case.

4.2.1 Optimal Solution

The social planner's problem is very similar to that in the previous chapter, except that the planner now also has to choose a time path for the control variable *u*. In addition he has to take into account the development of the new state variable *A*. Then the problem can be formalized as:

$$
\begin{aligned}
(4.3)\qquad & \max_{c_M,\, n,\, u} \int_0^{\infty} \frac{\left[(A^{\eta}(un)^{\alpha})^{\gamma} c_M^{1-\gamma} \right]^{1-\sigma}}{1-\sigma} e^{-\rho t} dt \\
& \text{s.t.} \quad \dot{K} = MK^{1-\alpha}(1-n)^{\alpha} - c_M \\
& \text{and} \quad \dot{A} = A\delta(1-u).
\end{aligned}
$$

The current-value Hamiltonian is given by

$$
(4.4)\quad H_c = \frac{\left[(A^{\eta}(un)^{\alpha})^{\gamma} c_M^{1-\gamma} \right]^{1-\sigma}}{1-\sigma} + \theta_1 (MK^{1-\alpha}(1-n)^{\alpha} - c_M) + \theta_2 A\delta(1-u).
$$

It has now seven solution equations:

$$
(4.5)\quad \frac{\partial H_c}{\partial c_M} = (1-\gamma)\left[(A^{\eta}(un)^{\alpha})^{\gamma} c_M^{1-\gamma} \right]^{1-\sigma} c_M^{-1} - \theta_1 = 0
$$

---

6. Note that the different specifications do indeed lead to different testable implications. In the first case the growth rate of per-capita food consumption depends on the *growth rate* of human capital while in the second case it depends on the *level* of human capital in agriculture. In a recent paper *Benhabib* and *Spiegel* (1994) have tested these differences empirically as possible determinants for the growth rate of per-capita income in a cross-section of countries. They find that the level effect of human capital in production is insignificant while the growth effect of human capital on total factor productivity is significant.

$$(4.6)\quad \frac{\partial H_c}{\partial n} = \gamma\alpha\left[\left(A^{\eta}(un)^{\alpha}\right)^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} n^{-1} - \alpha\theta_1 M K^{1-\alpha}(1-n)^{\alpha-1} = 0$$

$$(4.7)\quad \frac{\partial H_c}{\partial u} = \gamma\alpha\left[\left(A^{\eta}(un)^{\alpha}\right)^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} u^{-1} - \theta_2 A\delta = 0$$

$$(4.8)\quad \frac{\partial H_c}{\partial \theta_1} = \dot{K} = MK^{1-\alpha}(1-n)^{\alpha} - c_M$$

$$(4.9)\quad \frac{\partial H_c}{\partial \theta_2} = \dot{A} = A\delta(1-u)$$

$$(4.10)\quad \dot{\theta}_1 = \theta_1\rho - \frac{\partial H_c}{\partial K} = \theta_1\rho - \theta_1(1-\alpha)MK^{-\alpha}(1-n)^{\alpha}$$

$$(4.11)\quad \dot{\theta}_2 = \theta_2\rho - \frac{\partial H_c}{\partial A} = \theta_2\rho - \theta_2\delta(1-u) - \gamma\eta\left[\left(A^{\eta}(un)^{\alpha}\right)^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} A^{-1}$$

To be an optimal solution, the controls c_M, u, and n must again be chosen in a way that satisfies the boundary conditions. These are: (i) two initial values A_0 and K_0 as well as (ii) two transversality conditions $\lim_{t\to\infty} e^{-\rho t}\theta_1 K = 0$ and $\lim_{t\to\infty} e^{-\rho t}\theta_2 A = 0$.[7]

In addition the sufficiency conditions must be satisfied. However, we cannot show as we did in the last chapter, that the sufficiency conditions are always met. For the problem (4.3) *Mangasarian's* sufficiency conditions are not satisfied. However, these conditions are rather strong and could, in principle, be replaced by *Arrow's* condition which is weaker (*Berck* and *Sydsæter*, 1991). The latter, though, cannot be derived from the equations (4.4) - (4.7) due to the problem's non-linearity in n and $(1-n)$. Therefore we have to assume that the derived time paths for u, n, c_M, K, and A indeed solve problem (4.3). In the light of sufficiency of the basic problem from chapter 3 and the well-behaved utility and production function for goods this does not seem to be a problematic assumption.

Next, we consider the steady-state equilibrium. The growth rates of capital and per-capita consumption of the widgets can be obtained like above as:

7. For the justification of these transversality conditions cf. the discussion in chapter 3.

$$(4.12)\quad \left(\frac{\dot{c}_M}{c_M}\right)^* = \left(\frac{\dot{K}}{K}\right)^* = \frac{\mu}{\alpha}.$$

Again the growth rate of widget consumption depends only on technical progress in industry, not in any way on the outcome of agriculture.

Now consider u^*, the steady-state fraction of labor in actual food production. Inserting equation (4.7) into (4.11) yields

$$\frac{\dot{\theta}_2}{\theta_2} = \rho - \frac{\eta}{\alpha}\delta u - \delta(1-u).$$

Differentiating equation (4.7) with respect to time, equating the result to the above equation and rearranging the outcome leads to the steady-state value for u:

$$(4.13)\quad u^* = \frac{\alpha\left(\rho - (1-\sigma)\left(\gamma\eta\delta + (1-\gamma)\frac{\mu}{\alpha}\right)\right)}{\eta\delta\left(1-\alpha\gamma(1-\sigma)\right)}$$

The steady-state value for u also leads to the growth rate of (per-capita) food consumption. Differentiating the agricultural production function (4.1) and making use of equation (4.13) leads to the following steady-state value:

$$(4.14)\quad \left(\frac{\dot{c}_A}{c_A}\right)^* = \frac{\eta\delta - \alpha\left(\rho - (1-\gamma)(1-\sigma)\frac{\mu}{\alpha}\right)}{1-\alpha\gamma(1-\sigma)}$$

The growth rate of food production not only depends upon parameters describing agriculture but also, among other things, on μ, the rate of industrial technical progress. This is a new asymmetry in the dual economy model which, however, is caused by the hybrid character of the model with exogenous technical progress in industry and endogenous technical progress in agriculture. We will discuss the consequences in detail below.

The steady-state level of u is also necessary to calculate parameter restrictions for ρ from the transversality conditions. We show in appendix A.4 that both transversality conditions imply the same restriction, namely:

$$(4.15)\quad \rho > (1-\sigma)\gamma\eta\delta + (1-\sigma)(1-\gamma)\frac{\mu}{\alpha}$$

As a further restriction the values for u^* must lie in the interval (0, 1) since u is a bounded control. We show in appendix A.5 that this is guaranteed if the following condition is met:

$$(4.16)\quad (1-\sigma)\gamma\eta\delta < \rho - (1-\gamma)(1-\sigma)\frac{\mu}{\alpha} < \frac{\eta\delta}{\alpha}$$

Condition (4.16) demands that the efficiency of research or human capital accumulation δ must neither be too small nor too large. Parameter constellations violating this condition imply steady-state values for u which are simply not feasible. Note that the left inequality is again transversality condition (4.15).

It remains the steady-state value for n, the fraction of labor in agriculture. This is derived in appendix A.6 from the system's solution equations and (4.13) as:

$$(4.17)\quad n^* = \frac{\gamma\left(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma+\alpha^2\gamma)\frac{\mu}{\alpha}+\mu\right)}{\rho-(1-\sigma)\gamma\eta\delta+\sigma(1-\gamma+\alpha\gamma)\frac{\mu}{\alpha}+(1-\alpha)(1-\sigma)\gamma^2\mu}$$

We show in appendix A.7 that the transversality condition (4.15) is sufficient to ensure parameter values in the interval (0, 1) for n^*.

This concludes the description of the endogenous growth model's steady-state. It is given by equations (4.12), (4.13), (4.14), and (4.17). In addition, condition (4.16) has to be met for this equilibrium to be feasible. Note that the outcome is not a pure endogenous growth steady-state since technical progress in industry is still exogenous. However, even with $\mu = 0$ the economy does not stagnate since agricultural output grows without bounds, and the division of labor between the two sectors is well determined.

By introducing a further state and control variable compared to chapter 3, we might have changed the model's stability properties. Therefore they have to be analyzed again. As before, we check the model's local stability at the steady-state. The method is the same as in section 3.1.2. above, except that there are now three controls (n, u, and c_M) as well as two state variables (A and K). With three equations the steady-state Jacobian becomes rather large and no algebraic solution for the stability condition can be found. Therefore we analyze the stability properties first algebraically for logarithmic utility (i.e., $\sigma = 1$) and subsequently numerically for more general values of σ. We start by defining two variables, a control-like variable $z_1 = c_M / K$ and a state-like variable $z_2 = M / K^{\alpha}$. By equation (4.12) both are constant in the steady-state. No modifications of the third and fourth variables, n and u, are necessary.

Combination of equations (4.5) and (4.6) leads to an expression for z_1 that defines the value for the control-like variable z_1 depending on z_2 and n:

$$z_1 = \frac{(1-\gamma)}{\gamma} z_2 n (1-n)^{\alpha-1}$$

This reduces the model by one dimension since z_1 can be calculated at every instance from z_2 and n. In appendix A.8 we derive the remaining differential equations for u, n, and z_2 that describe the economy's development towards the steady-state equilibrium. For the special case that $\sigma = 1$ these differential equations simplify to:

$$\begin{aligned} \frac{\dot{n}}{n} &= \frac{(1-n)}{(1-\alpha n)}\left[-\rho-\mu+(1-\alpha)\, n z_2 (1-n)^{\alpha-1}\frac{(1-\gamma)}{\gamma}\right] \\ \frac{\dot{u}}{u} &= -\frac{\alpha\rho-\eta\delta u}{\alpha} \\ \frac{\dot{z}_2}{z_2} &= \mu-\alpha z_2 (1-n)^{\alpha-1}\left(\frac{\gamma-n}{\gamma}\right) \end{aligned} \tag{4.18}$$

The steady-state values for n, u, and z_2 are obtained by setting all equations in (4.18) equal to zero and solving for z_2, n, and u. Three solutions exist, of which one is interior and the others are corner solutions. The corner solutions ($n^* = 1$, $n^* = 0$) imply again a collapse of the dual economy and furthermore violate the transversality condition. We therefore consider only the interior solution which is characterized by:

$$\begin{aligned} n^* &= \frac{\gamma(\rho+\mu)}{\rho+(1-\gamma+\alpha\gamma)\dfrac{\mu}{\alpha}} \\ u^* &= \frac{\alpha\rho}{\eta\delta} \\ z_2^* &= \frac{\left(\rho+\dfrac{\mu}{\alpha}\right)\left[\dfrac{(1-\gamma)(\alpha\rho+\mu)}{\alpha\rho+(1-\gamma+\alpha\gamma)\mu}\right]^{-\alpha}}{(1-\alpha)} \end{aligned} \tag{4.19}$$

This equilibrium is stable if the system's Jacobian evaluated at the steady-state has two eigenvalues with positive real parts (since there exist two control variables u and n) and one with a negative real part. Multiple equilibria would be characterized by two or three eigenvalues with negative real parts, and the equilibrium would be unstable if all eigenvalues had positive real parts.

Since the steady-state Jacobian J^* is now a 3 x 3 matrix, its eigenvalues are given by the solution to the following characteristic equation:

$$-r^3 + \mathrm{Tr}J^* r^2 - \mathrm{B}J^* r + \mathrm{Det}J^* = 0$$

where $\mathrm{Tr}J^*$ is the trace of the evaluated Jacobian J^* and $\mathrm{Det}J^*$ its determinant. $\mathrm{B}J^*$ is the sum of its principal minors of order two. Instead of calculating the eigenvalues by solving the characteristic equation, we can again make use of the Routh-Hurwitz theorem. It states that for 3 variables the number of roots with positive real parts is equal to the number of variations of sign in the following scheme:

$$-1, \qquad \mathrm{Tr}J^*, \qquad -\mathrm{B}J^* + \frac{\mathrm{Det}J^*}{\mathrm{Tr}J^*}, \qquad \mathrm{Det}J^*$$

The terms can be obtained for the simple case that $\sigma = 1$ as:

$$\mathrm{Det}J^* = \frac{\rho(\mu+\rho)(\mu+\alpha\rho)((1-\gamma+\alpha\gamma)\mu+\alpha\rho)}{(\alpha-1)((1-\gamma)\mu+\alpha\gamma(1-\alpha)\mu+\alpha(1-\alpha\gamma)\rho)}$$

$$-\mathrm{B}J^* + \frac{\mathrm{Det}J^*}{\mathrm{Tr}J^*} = \frac{(\mu+\alpha\rho)((1-\gamma+\alpha\gamma)\mu+\alpha\rho)}{(1-\alpha)((1-\gamma)\mu+\alpha\gamma(1-\alpha)\mu+\alpha(1-\alpha\gamma)\rho)}$$

$$-\frac{(\mu+\rho)\alpha(\mu+\rho)(\mu(\gamma-1-2\alpha\gamma+\alpha^2\gamma)+\alpha\rho)}{(1-\alpha)((1-\gamma)\mu+\alpha\gamma(1-\alpha)\mu+\alpha(1-\alpha\gamma)\rho)} + (\mu^2+\mu\rho-\rho^2)$$

$$\mathrm{Tr}J^* = 2\rho$$

It is obvious that by the assumptions about the parameter values $\mathrm{Det}J^* < 0$ and also $\mathrm{Tr}\,J^* > 0$. Thus, no matter what sign the last term involving $\mathrm{B}J^*$ has, the scheme is either – + – – or – + + –. In both cases there are two sign changes and thus two positive roots. The system is saddle path stable.

Although this solution gives some insight into the stability issues it is not very general because $\sigma = 1$. We therefore conduct again numerical calculations of the system's eigenvalues for more general values for σ using *Mathematica's* Eigenvalue routine. We choose a high and a low value for μ and γ, four different values for σ, and vary δ over the range from 0.025 to 1.0. This range for δ includes the value chosen by *Lucas* (1988), namely $\delta = 0.05$. In his model this parameter value implies a steady-state fraction of labor in production of 0.82. Like above, we set $\alpha = 0.07$ and $\rho = 0.05$. The calculations are summarized in table 9.

Table 9: Saddle Path Stability for Endogenous Growth Model

σ	γ	μ	δ^{a}	σ	γ	μ	δ
10	0.8	0	0.05 – 1.0	1	0.8	0	0.05 – 1.0
		0.02	0.075 – 1.0			0.02	0.05 – 1.0
	0.4	0	0.05 – 1.0		0.4	0	0.05 – 1.0
		0.02	0.15 – 1.0			0.02	0.05 – 1.0
5	0.8	0	0.05 – 1.0	0.5	0.8	0	0.05 – 1.0
		0.02	0.075 – 1.0			0.02	0.05 – 1.0
	0.4	0	0.05 – 1.0		0.4	0	0.05 – 1.0
		0.02	0.1 – 1.0			0.02	0.05 – 1.0

a. Some δ-values missing due to violation of condition (4.15).

Table 9 shows that the analytical result can be extended to a broad range of plausible parameter values. This does not exclude multiple equilibria for other, unusual combinations of parameter values, though.[8] This result is an interesting supplement to the discussion when indeterminacies can arise in endogenous growth models which has been mentioned in section 3.1.2. While *Boldrin* and *Rustichini* (1994) have found multiple equilibria under rather mild assumptions in a two-sector economy consisting of a consumption good sector and a capital good sector, the model presented here shows uniqueness of the steady-state equilibrium for a large range of parameter values. Possibly capital and human capital accumulation have to occur in the same sector to generate multiple equilibria which is not the case in the model derived here, where capital is only accumulated in industry and human capital or technical knowledge only in agriculture. It is, though, a feature in most other endogenous growth models, many of which showing multiple equilibria. These models focus on industry as capital intensive *and* human capital intensive sector.

8. Experiments with other parameter combinations have shown, however, that combinations which yield other signs for the eigenvalues violate condition (4.16).

Also multiple growth paths do only occur in the sense discussed in the previous chapter. Although a further state variable has been introduced into the model, this had not the same consequences as in *Lucas'* (1988) model where it made the growth path depend on a country's history. As before, the state variable A does not appear in the transformed differential equations given by (4.19). This is also due to the fact that the state variables here (A, K) do not enter the same production function which they do in the standard endogenous growth model.

4.2.2 Market Outcome

We now consider the market outcome when externalities of research are not taken into account and compare the solution to the optimal outcome. We therefore consider not anymore the optimization problem of a social planner but rather that of a single individual who has to make decisions about c_M, u, and n. Since the number of individuals is normalized to unity, the decision problem is similar to (4.3) apart from one small difference: To include explicitly the externality, the agricultural production function (4.1) is now written as

$$(4.20)\quad Y_A = \bar{A}^{\eta_1} A^{\eta_2} (un)^{\alpha}, \qquad 0 < \alpha < 1$$

where A^{η} from equation (4.1) is now decomposed into $\bar{A}^{\eta_1}$, the part of technology or human capital in agriculture which is taken as exogenous by individuals, and A^{η_2}, the part of the effect that is taken into account. As before, $\eta_1 + \eta_2 = \eta$.

The resulting outcome of the maximization problem is not any more an optimal outcome because agents misperceive the effects of their research or human capital accumulation decision. Therefore the notion of a dynamic equilibrium becomes more complicated as *Lucas* (1988) has pointed out. A dynamic equilibrium requires not only that markets clear in every period but also that agents make correct forecasts about the variables (here $\bar{A}$) so that they do not regret their decisions once they observe the actual values. This idea of a dynamic equilibrium bears some resemblance to a rational expectations equilibrium (cf. *Hahn*, 1987). For our model such an equilibrium requires that $A = \bar{A}$.

Letting the individuals solve the problem (4.3) with equation (4.20) instead of (4.1) as production function modifies the Hamiltonian slightly. Maximizing it with $\bar{A}$ taken as exogenous and then substituting A for $\bar{A}$ into the outcome again leads to seven solution equations of which the first six are the same as (4.5) – (4.10). Only equation (4.11) changes into

$$(4.21)\quad \dot{\theta}_2 = \theta_2\rho - \frac{\partial H_c}{\partial A} = \theta_2\rho - \theta_2\delta(1-u) - \gamma\eta_2\left[(A^{\eta}(un)^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} A^{-1}.$$

Note that the only difference between both equations is η_2 instead of η in the third term.

This change does not affect the growth rates of capital or per-capita widget consumption as one can see from the derivation of equation (4.12) above. However, it alters the steady-state fraction of agricultural labor in production, u^*, into[9]

$$(4.22)\quad u^{**} = \frac{\alpha\left(\rho - (1-\sigma)\left(\gamma\eta\delta + (1-\gamma)\frac{\mu}{\alpha}\right)\right)}{\delta\left(\eta_2 - \eta\alpha\gamma(1-\sigma)\right)}.$$

Note that an additional condition for non-negativity of u^{**} is that $\eta_2 - (1-\sigma)\eta\alpha\gamma > 0$: the part of the productivity increasing effect taken into account by the agents must not be too small. If this condition is met, one can see from comparing equation (4.13) to equation (4.22) that $u^{**} > u^*$. Agents choose a lower fraction of labor in research than optimal. This influences the food growth rate which due to

$$\frac{\dot{c}_A}{c_A} = \eta\delta(1-u)$$

now decreases.

Consider now the market outcome for the division of labor between industry and agriculture. Equation (4.17) changes into:

$$(4.23)\quad n^{**} = \frac{\gamma\left(\rho - (1-\sigma)\gamma\eta\delta - (1-\sigma)\left(1-\gamma+\alpha^2\gamma\frac{\eta}{\eta_2}\right)\frac{\mu}{\alpha} + \mu\right)}{\rho - (1-\sigma)\gamma\eta\delta + \left(1-\gamma+\frac{\alpha\gamma\eta}{\eta_2}\right)\frac{\sigma\mu}{\alpha} + \gamma\mu\left(\frac{\eta_2-\eta}{\eta_2}\right) + \frac{(1-\alpha)(1-\sigma)\eta\mu\gamma^2}{\eta_2}}$$

9. We characterize the steady-state values for the market solution by a two-star superscript.

The difference to equation (4.17) is the appearance of an "externality wedge" η/η_2. This term is unity if the full externality is taken into account. Thus, the steady-state division of labor between both sectors depends on the size of the externality. The smaller η_2, the effect of research taken into account by the private sector, the larger this wedge. We show in appendix A.9 that $\partial n^{**}/\partial(\eta/\eta_2)$ is positive for $\sigma < 1$, zero for $\sigma = 1$, and negative for $\sigma > 1$. These signs are conditional on $\mu > 0$. For $\mu = 0$ the externality wedge has no influence on n^{**} as one can also see from equation (4.23). If, for example $\sigma > 1$, as we have assumed above, the larger the externality (and thus this wedge) is, the smaller the fraction of labor in agriculture.

Therefore the existence of externalities is not only responsible for lower growth rates of food consumption than optimal but can also result in over-industrialization. This might look like a strange result at first sight. After all, we know that developing countries are usually under- and not over-industrialized. However, the resulting over-industrialization is a consequence of the neoclassical assumption of full-employment. If this assumption were released, for example along the lines of the *Harris/Todaro* model mentioned in chapter 2, the force leading to over-industrialization in this model would still prevail: there would be a tendency in this economy to leave the agricultural sector in favor of industry since farmers underestimate the possible gains from staying in agriculture and improving agricultural productivity. This flight into the city is something we do indeed observe in developing countries: according to *Williamson* (1988), the urban share of the Third World's population rose from 9.3 to 28 percent between 1925 and 1975. The model would therefore have to be interpreted as stating that a reduction of the externality wedge could lessen the desire to migrate into the cities and therefore alleviate the pressure on the cities from rural-urban migration.

Table 10 shows what consequences the externality can have. For the calculations of the effects we have chosen $\rho = 0.05$, $\alpha = 0.7$, and $\eta = 1$ of which $\eta_2 = 0.75$ is taken into account.[10] The choice of this externality gap is of course rather arbitrary. However, estimates on the return to investment in agricultural research and

10. Other values for η_2 lead to similar results.

human capital formation vary so much that no uniform value can be obtained from this work.

Table 10: Effects of a 25% Externality

σ	γ	δ	μ	n^*	u^*	$(\dot{c}_A/c_A)^*$	n^{**}	u^{**}	$(\dot{c}_A/c_A)^{**}$
10	0.8	0.2	0	0.8	0.863	0.027	0.8	0.901	0.020
			0.02	0.795	0.893	0.021	0.794	0.932	0.014
		0.1	0	0.8	0.892	0.011	0.8	0.931	0.007
			0.02	0.791	0.952	0.005	0.789	0.993	0.001
	0.4	0.2	0	0.4	0.766	0.047	0.4	0.834	0.035
			0.02	0.393	0.919	0.016	0.391	0.989	0.002
		0.1	0	0.4	0.815	0.018	0.4	0.878	0.012
			0.2	N.A.[a]	N.A.[a]	N.A.[a]	N.A.[a]	N.A.[a]	N.A.[a]
5	0.8	0.2	0	0.8	0.745	0.051	0.8	0.808	0.038
			0.02	0.794	0.770	0.046	0.793	0.834	0.033
		0.1	0	0.8	0.799	0.020	0.8	0.866	0.013
			0.02	0.790	0.849	0.015	0.789	0.919	0.008
	0.4	0.2	0	0.4	0.611	0.078	0.4	0.693	0.061
			0.02	0.391	0.724	0.055	0.390	0.821	0.036
		0.1	0	0.4	0.693	0.031	0.4	0.786	0.021
			0.02	0.387	0.920	0.008	N.A.[a]	N.A.[a]	N.A.[a]
1	0.8	0.2	0	0.8	0.175	0.165	0.8	0.233	0.153
			0.02	0.781	0.175	0.165	0.781	0.233	0.153
		0.1	0	0.8	0.35	0.065	0.8	0.467	0.053
			0.02	0.781	0.35	0.065	0.781	0.467	0.053
	0.4	0.2	0	0.4	0.175	0.165	0.4	0.233	0.153
			0.02	0.373	0.175	0.165	0.373	0.233	0.153
		0.1	0	0.4	0.35	0.065	0.4	0.467	0.053
			0.02	0.373	0.35	0.065	0.373	0.467	0.053

a. This parameterization violates condition (4.16).

The numerical simulations show that the influence of the externality wedge on the split of labor between the two sectors is rather low, if it exists at all. However, the influence on the engagement in research or human capital accumulation is relatively high, leading to a reduction in the growth rate of food consumption of up to 1.5 percentage points. While this number is rather arbitrary, it does show that the effects of an externality on the growth rate of food production can indeed be large.

4.3 Economic Policy, Preferences, and Development

We can now turn again to the influences of parameter changes on the steady-state as well as on the short-run transition dynamics. As in the previous chapter, we start with a comparative static analysis of the steady-state outcome and discuss several economic policies within this context. As second step we solve the model numerically to study the length of the transitional period as well as the influence of economic policies on it.

4.3.1 Long-Run Effects

We start by calculating the analytical derivatives of u^* and n^* with respect to economic policies and preference changes. Since $\lambda = 0$ in this chapter, the remaining policies are changes in the rate of technical progress in industry, μ, as well as changes in δ. Since the latter have different interpretations depending on the question whether agricultural productivity is increased by research or by human capital accumulation, we will interpret the outcome as we go along. In addition we investigate the consequences from reducing the externality wedge. In the light of the results from previous chapter also changes of σ and γ are of interest. Besides we derive the influence of η.

The signs of the derivatives of u^* are obtained in appendix A.10 and compiled in table 11. Unfortunately changes of γ are again ambiguous and therefore not reported here. Apart from effects emanating in the agricultural sector (changes in research efficiency δ and output elasticity of technical knowledge η) these derivatives show that there exists an indirect effect of the rate of technical progress in industry on the allocation of labor to research: an increase of μ influences the fraction of agricultural labor in research $(1 - u^*)$. It is decreased for $\sigma > 1$, increased for $\sigma < 1$, and remains constant for logarithmic utility. The growth rate of per-capita food consumption will be influenced in the same direction via this mechanism as one can see from equation (4.14).

Table 11: Derivatives of u^*

Derivatives	$\sigma < 1$	$\sigma = 1$	$\sigma > 1$
$\partial u^* / \partial \mu$	(−)	0	(+)
$\partial u^* / \partial \delta$	(−)	(−)	(−)
$\partial u^* / \partial \eta$	(−)	(−)	(−)
$\partial u^* / \partial \sigma$	(+)	(+)	(?)

The growth rate of food consumption also rises if η or δ are increased. These are effects that one would expect. An increase in δ raises the efficiency of research and according to equation (4.13) also the fraction of agricultural labor in research. An increase in η, the output elasticity of technology or human capital, raises the marginal output increases from each new invention or additional unit of human capital. In addition, by equation (4.13) an increase in η also raises the research or human capital accumulation effort. Both effects lead to a larger growth rate of agricultural technology or human capital and also of food output.

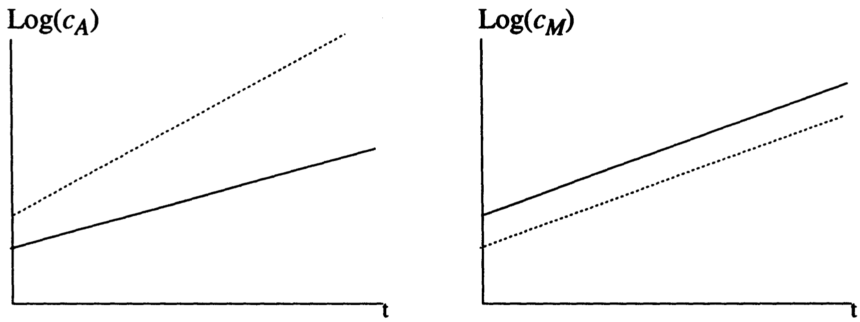
Signs of the derivatives of n^* are derived in appendix A.11 and summarized in table 12.

Table 12: Derivatives of n^*

Derivatives	$\sigma < 1$	$\sigma = 1$	$\sigma > 1$
$\partial n^* / \partial \mu$	(−)	(−)	(−)
$\partial n^* / \partial \delta$	(−)	0	(+)
$\partial n^* / \partial \eta$	(−)	0	(+)
$\partial n^* / \partial \sigma, \mu > 0$	(+)	(+)	(+)
$\partial n^* / \partial \sigma, \mu = 0$	0	0	0

We can now consider both effects of economic policies together. Turning to an increase in δ first, the signs of the derivatives of n^* and u^* with respect to δ imply a smaller u^* and a larger n^* (if $\sigma > 1$ as we still assume). Thus, the fraction of labor in the agricultural sector increases. Within this sector the fraction of agricultural labor in schooling or research rises which, due to equation (4.9), increases the growth rate of its output, A. Therefore per-capita consumption of food also grows faster. The growth rate of c_M, though, remains unchanged as equation (4.12) shows. Thus, in industry an increase in δ only causes a level effect, while it induces both, level and growth effects, in the agricultural sector. Note that the initial effect on food consumption is ambiguous. The agricultural labor force as a whole increases but a larger fraction of labor now engages in schooling or research. One possible outcome is depicted in figure 8.

Figure 8: Increasing Research Efficiency in Agriculture



Legend: solid line = low δ; dotted line = high δ.

An increase in δ raises the marginal utility from working in research or human capital accumulation instead of working in food production. This disequilibrium in equation (4.7) leads to a shift of agricultural labor into research until the marginal utility of working in food production has risen sufficiently to restore the equilibrium. However, now equation (4.6) which balances the marginal utilities from using labor in either sector becomes unbalanced. To raise the marginal utility from working in industry and to decrease that from working in agriculture, a larger fraction of labor has to be employed in the agricultural sector.[11]

For a policy maker increasing the efficiency of research could mean anything that improves the productivity of labor in this activity, from an improved flow of information into research to decreased X-inefficiencies (*Leibenstein*, 1966) in governmental research institutions. However, the model's feature that the output of research is immediately productive points to a second implication: increasing the efficiency of research can as well mean increasing the research output that *is* immediately usable in agricultural production. The same is of course valid for

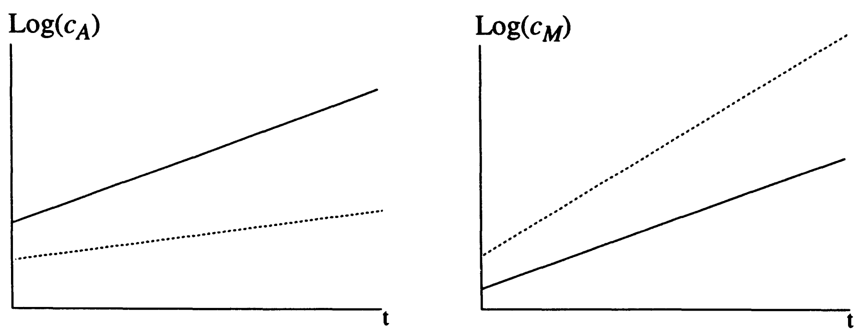
11. Note that this second effect depends on σ. For $\sigma = 1$ it disappears while it works in the opposite direction for $\sigma < 1$.

human capital accumulation: the efficiency of schooling can be increased by raising that fraction of the curriculum which contains knowledge directly relevant for agricultural production.

Equivalent to the previous chapter, an increase in δ does not lead to a larger degree of industrialization. This is again due to the feature that in this model all additional food is consumed while in classical dual economy models mentioned in chapter 2 the income elasticity of food demand is less than unity. Therefore an increase in the rate of technical progress leads to labor migration into industry.

Next consider an increase in μ, the rate of technical progress in industry. As before we focus on $\sigma > 1$. According to the derivatives of u^* and n^* with respect to μ an increase in the variable leads to an increase in u^*. Thus, a smaller fraction of agricultural labor engages in research or human capital accumulation which decreases the growth rate of food consumption. The increase in μ also causes a decrease in n^*, and thus a shift of labor into industry. Due to the higher μ also the growth rate of c_M rises. The reactions are depicted in figure 9.

Figure 9: Increasing Technical Progress in Industry



Legend: solid line = low μ; dotted line = high μ.

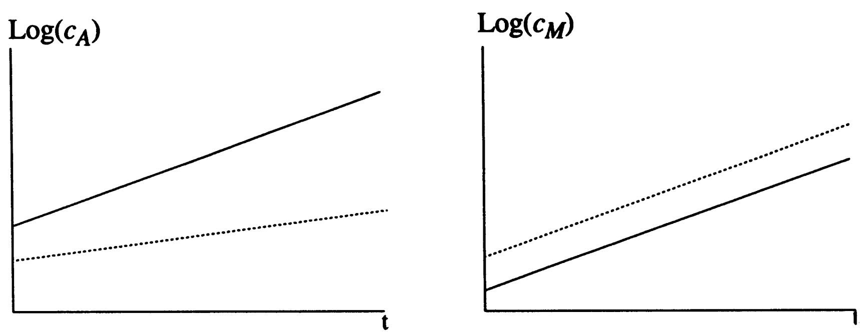
The intuition behind this behavior is the following: an increase in μ makes the time path of M steeper. Taking equation (4.6) in growth rates, the right-hand side increases ceteris paribus: marginal utility from working in industry rises faster than that from working in agriculture which cannot constitute an equilibrium. Due to equation (4.12) also the growth rates of K and c_M rise. While the first increase tends to rebalance the growth rates, the second does the opposite if $\sigma > 1$. A possibility to restore the equilibrium is a decrease in the growth rate of A which can be achieved by employing less labor in research. Thus, *u* has to rise. This reaction, however, decreases the left-hand side of equation (4.6) in levels, the marginal utility from working in agriculture. Hence, labor migrates to industry to rebalance marginal utilities.

Therefore an increase in the rate of technical progress in industry has exactly the opposite effect of an increase in the rate of technical progress in agriculture by raising research or schooling efficiency. While the latter boosts the agricultural sector, the former raises the fraction of labor as well as the rate of output growth in industry. But in addition an increase in μ reduces the resources devoted to agricultural research or human capital accumulation and thus also decreases the growth rate of food production – probably not what the policy maker intended. The reason for this asymmetry is that technical progress in agriculture is endogenous while it is exogenous in industry. If both were endogenous, this influence of technical progress in industry on technical progress in agriculture would probably also be observable in the other direction. The existence of this interdependence shows the usefulness of our endogenous growth model, since such an interdependence is not captured by the exogenous growth model. As we will see below, the effect can be quite large.

While one could endogenize technical progress in manufacturing in the same way as in the agricultural sector, there are fundamental differences between technical progress in these sectors which might make it necessary to choose a different approach. For example, the fraction of industrial research conducted within private firms is much larger than in agriculture where government research institutions are very important. Hence, one should take into account important problems associated with private research like patent protection, creative destruction, etc., very much like the innovation-driven growth literature mentioned in chapter 2 does. Since this is beyond the scope of this study, the determinants of industrial technical progress are not discussed further at this place.

The policies considered so far are based on the optimal solution but have the same effects in case of a market solution. In this situation a further policy is possible when agents do not recognize the full dynamic effect of research or human capital accumulation. The consequences of such externalities have been derived in the last section. With $\eta / \eta_2 \geq 1$ denoting the externality wedge it has been shown that for $\sigma > 1$ a larger wedge, that is, a larger difference between total effect and the effect taken into account, leads to a smaller fraction of labor in agriculture and in research or human capital accumulation. The latter decreases the growth rate of food consumption. These effects are depicted in figure 10.

Figure 10: Externality Effects



Legend: solid line = optimal solution; dotted line = market solution with externality.

This outcome is caused in the following way: According to equation (4.21) the externality c.p. leads to a higher growth rate of θ_2, hence a flatter growth path of this co-state variable (recall that θ_2 decreases over time). Individuals think that they do not loose too much by working now and not engaging in schooling or research since they underestimate the gain from this activity. This unbalances equation (4.7) in growth rates which can only be rebalanced by lower growth rates of A: the fraction of labor in agricultural research $(1 - u)$ falls, u rises. Thus, by equation (4.6) the fraction of labor in agriculture is smaller than optimal. As consequence, such an economy has a larger fraction of labor in industry and grows slower than optimal.

Knowing these causalities, the government could try to influence the control variable responsible for the inferior outcome, namely u. With subsidies or taxes it could increase the fraction of agricultural labor in research or human capital accumulation. With respect to schooling the model implies that some compulsory level of schooling might be a good policy. Parents would tend to consider only the influence of their children's schooling on the own farm production, not the general economic effect resulting from a higher average level of schooling. They would probably also value the lack of their children's work force rather high, this even more so if they live close to subsistence.

The second possible economic policy would be to decrease the externality wedge η / η_2. A policy to achieve that goal in research would be some kind of patent protection that allows the developer of a new technique to reap a larger share of the productivity gains from new production techniques. Another possibility for the government would be to initiate research projects itself while making use of agricultural labor to conduct them.[12] With respect to human capital accumulation there is no such simple policy to reduce the externality wedge. Comparing the two possible policies to overcome the inferior market solution, an increase of the fraction of labor in schooling or research seems to be easier to achieve than a decrease of the externality wedge.

Last, consider a change in σ. While an increase in σ increases n^*, the effect on u^* is ambiguous for $\sigma > 1$. Thus, the larger σ, the less industrialized is a country which is the same effect as in the previous chapter. Again this effect seems to be rather fragile since it disappears with $\mu = 0$.

We now calculate again numerical values for the steady-states to discuss the quantitative effects of economic policies. These calculations also yield information about the influence of σ on u^* as well as about the influence of changes in γ on the steady-state. The parameters are the same as in previous chapter: $\alpha = 0.7$, $\rho = 0.05$, $\eta = 1$. For γ, μ, and δ a high and low value are supplied where δ has be chosen to yield "reasonable" growth rates at least for some parameterizations. For σ the values 10, 5, and 1 are chosen. The results are reported in table 13.

The outcome confirms previous results: economic policy alone cannot explain the observed shift of labor from agriculture into industry (our first stylized fact); the introduction of endogenous technical progress did not change this result. The consequences of an increase in δ are rather small. Doubling this efficiency measure from 0.1 to 0.2 only influences the split of labor between the two sectors in the magnitude of tenths of percentage points. As in chapter 3, changes in γ, the weight of food in the utility function, are the only force strong enough to replicate

12. Most of agricultural research is indeed primarily a public-sector activity. *Judd, Boyce,* and *Evenson* (1986) point out that typical public good problems are particularly acute in agricultural research. While chemical and mechanical inventions can be patented relatively easily, this is more difficult for biological inventions concerning plant breeding, phytopathology, entomology, agronomy, soil science, animal nutrition, etc.

the observed structural change. Again, changes in σ only have a rather small effect.

Table 13: Steady-States for Different Parameter Values

σ	γ	δ	μ	n^*	u^*	z_2^*	$(\dot{c}_A/c_A)^*$	$(\dot{c}_M/c_M)^*$	r^a
10	0.8	0.2	0	0.8	0.863	2.537	0.027	0	0.247
			0.02	0.795	0.893	2.869	0.021	0.029	0.286
		0.1	0	0.8	0.892	1.311	0.011	0	0.127
			0.02	0.791	0.952	1.643	0.005	0.029	0.165
	0.4	0.2	0	0.4	0.766	1.043	0.047	0	0.219
			0.02	0.393	0.919	1.376	0.016	0.029	0.291
		0.1	0	0.4	0.815	0.555	0.018	0	0.116
			0.02	N.A.[b]	N.A.	N.A.	N.A.	N.A.	N.A.
5	0.8	0.2	0	0.8	0.745	2.190	0.051	0	0.213
			0.02	0.794	0.770	2.507	0.046	0.029	0.249
		0.1	0	0.8	0.799	1.174	0.020	0	0.114
			0.02	0.790	0.849	1.491	0.015	0.029	0.150
	0.4	0.2	0	0.4	0.611	0.832	0.078	0	0.175
			0.02	0.391	0.724	1.111	0.055	0.029	0.235
		0.1	0	0.4	0.693	0.472	0.031	0	0.099
			0.02	0.387	0.920	0.751	0.008	0.029	0.160
1	0.8	0.2	0	0.8	0.175	0.514	0.165	0	0.05
			0.02	0.781	0.175	0.758	0.165	0.029	0.079
		0.1	0	0.8	0.35	0.514	0.065	0	0.05
			0.02	0.781	0.35	0.758	0.065	0.029	0.079
	0.4	0.2	0	0.4	0.175	0.238	0.165	0	0.05
			0.02	0.373	0.175	0.363	0.165	0.029	0.079
		0.1	0	0.4	0.35	0.238	0.065	0	0.05
			0.02	0.373	0.35	0.363	0.065	0.029	0.079

a. Interest rate r is given by the marginal product of capital $(1-\alpha)MK^{-\alpha}(1-n)^{\alpha}$.
b. This parametrization violates condition (4.16).

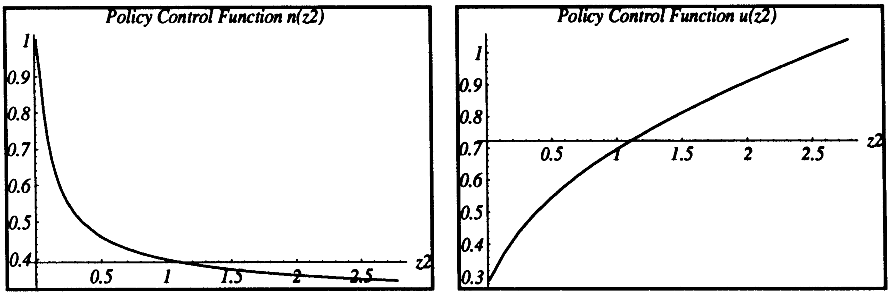
However, this order of magnitudes does not always persist with respect to changes in u^*. In some cases a change in μ, for example, can have larger effects on the steady-state level of u^* than a change in γ. There is no clear pattern, though, which effects are the strongest. In addition, the variation of u^* caused by changes in σ is rather large. While for $\sigma = 10$ individuals spend more than three fourths of their time in production, for $\sigma = 1$ this fraction reduces to one third or less. The latter magnitude does not seem to be very realistic. However, it shows clearly the effects of intertemporal substitution. If σ is low and therefore the intertemporal elasticity of substitution is large, the individuals use their chance of engaging in research or human capital accumulation to increase food consumption tomorrow. If this ability or willingness is less pronounced, they spend more of their time working in food production. As the calculations show, the consequences of these effects for the growth rates of food consumption are considerable.

4.3.2 Transitional Dynamics

We can now discuss the short-run dynamics of the endogenous growth model and the influences different parameters have on these dynamics. To be able to compare these simulations with those from chapter 3, we only discuss the optimal solution here. The dynamics for the market solution are not fundamentally different, though.

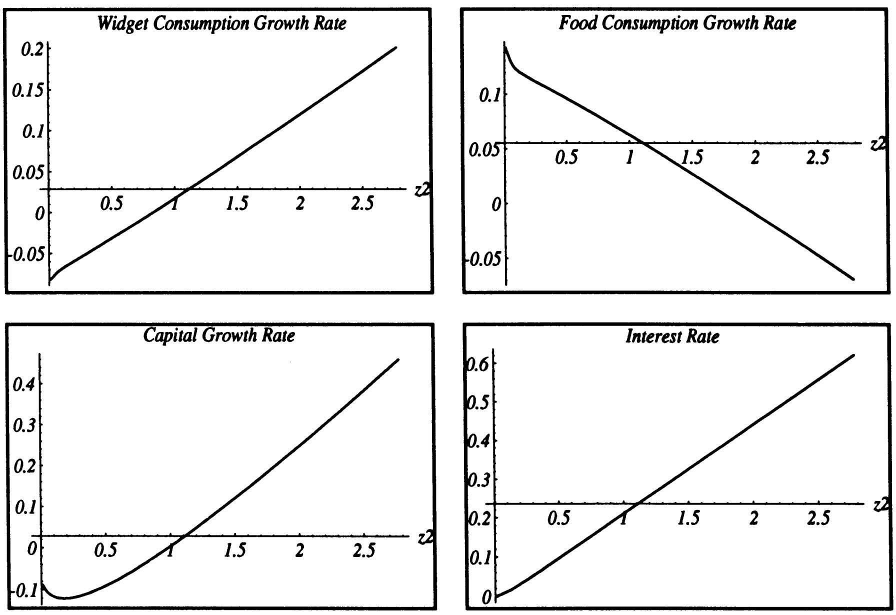
The questions we have in mind, are the same as in the previous chapter: How long do the transitional dynamics last? Is the model able to replicate the observed shift of labor into industry (first stylized fact) within the observed time-scale (second stylized fact)? Are the transitional dynamics realistic at all? And finally: What are the influences of economic policies on these dynamics? The methodology for simulating the transitional dynamics is also the same as in the previous chapter, namely the time-elimination method. The only difference is that there are now two control-like variables, n and u, together with the state-like variable z_2. The differential equations for these variables have already been derived in appendix A.8. These equations can be used to calculate control functions as set out in chapter 3. The resulting numerical control functions $n(z_2)$ and $u(z_2)$ are depicted in figure 11. The control-function $n(z_2)$ looks very much like in the exogenous growth model. The function $u(z_2)$ points to a restriction for the transition dynamics. For too high values of z_2 the optimal solution would require values of u larger than unity which is not feasible. Thus, z_2 must not be too high in the

Figure 11: Control Functions



beginning for a feasible equilibrium path to exist. The behavior of the remaining variables is shown in figure 12. The parameters for both figures are given as $\alpha = 0.7$, $\rho = 0.05$, $\sigma = 5$, $\gamma = 0.4$, $\eta = 1$, $\mu = 0.02$, and $\delta = 0.2$.

Figure 12: Transitional Behavior of Growth and Interest Rates



As before, figure 11 shows what values for the control variables a social planner has to choose for each value of the state-like variable z_2 given on the abscissa. Figure 12 shows growth rates of the major variables as well as the interest rate for the equilibrium paths towards the steady-state. The functional form for the policy function $n(z_2)$ is the same as in chapter 3. For z_2 less than its steady-state value, the planner has to choose a high value for n which can be decreased as z_2 raises towards its steady-state. Such a temporary rise in agricultural employment is accompanied by a fall in the fraction of agricultural labor engaged in food production as the plot for $u(z_2)$ shows.

Table 14: Duration of Transitional Dynamics

σ	γ	δ	μ	$z_2^* - 20\%$	$z_2^* - 10\%$	$z_2^* + 20\%$	$z_2^* + 5\%$
10	0.8	0.2	0	15.04	17.32	1.42	2.71
			0.02	12.97	14.91	N.A.[a]	N.A.[a]
		0.1	0	28.55	33.18	N.A.[a]	N.A.[a]
			0.02	21.74	25.25	N.A.[a]	N.A.[a]
	0.4	0.2	0	32.36	38.10	N.A.[a]	N.A.[a]
			0.02	23.74	27.89	N.A.[a]	N.A.[a]
		0.1	0	60.33	71.35	N.A.[a]	N.A.[a]
			N.A.[b]	N.A.[b]	N.A.[b]	N.A.[b]	N.A.[b]
5	0.8	0.2	0	16.68	19.22	1.83	3.33
			0.02	14.17	16.31	1.11	2.36
		0.1	0	30.57	35.54	4.84	8.05
			0.02	22.88	26.59	3.40	5.74
	0.4	0.2	0	35.19	41.48	6.64	10.87
			0.02	25.43	29.92	4.54	7.50
		0.1	0	61.61	72.89	12.55	20.35
			0.02	36.62	43.29	N.A.[a]	N.A.[a]
1	0.8	0.2	0	55.01	63.87	9.18	15.06
			0.02	33.89	39.44	5.52	9.13
		0.1	0	55.01	63.87	9.18	15.06
			0.02	33.89	39.44	5.52	9.13
	0.4	0.2	0	63.56	75.45	13.37	21.63
			0.02	39.32	46.65	8.00	13.04
		0.1	0	63.56	75.45	13.37	21.63
			0.02	39.32	46.65	8.00	13.04

a. For these parameterizations is the value for u greater unity in the first periods.
b. This parametrization violates condition (4.16).

We next calculate the duration of the transition dynamics for a variety of parameterizations which are given in table 14 together with the results. As before, the remaining parameter values are $\alpha = 0.7$, $\rho = 0.05$, and $\eta = 1$. The experiment is the same as in chapter 3. We assume that in the beginning z_2 is at $z_2^* \pm 90\%$ and we record the number of years until $z_2 = z_2^* \pm 20\%$ and $z_2 = z_2^* \pm 10\%$. Table 14 shows the results.

The first observation is that these durations are considerably smaller than in the model with exogenous growth. This reflects the fact that there is now an additional control variable which can be used to reach the steady-state equilibrium. This result casts some doubt on the conclusion drawn from the exogenous growth model that development could be understood as a transition between two steady-states in this model. To analyze this question further, we conduct below an experiment similar to that from previous chapter.

A second result is that increasing σ now decreases the transition duration, which is exactly the opposite result of that for the one-sector model obtained by *King* and *Rebelo* (1993) and even stronger than the ambiguity obtained in chapter 3. A hint for the causality of this effect is given by the dependence of u^* on σ. The higher the latter, the larger the former since individuals are less willing to forgo current consumption in favor of future consumption. Suppose that initially the state-like variable $z_2 = M / K^{\alpha}$ is higher than its steady-state level, which is the case if K grows below its steady-state growth path. To reach this path, capital growth has to accelerate during the transition period. This higher growth rate can be achieved by two means, by less consumption of widgets or by more labor in widget (and thus capital) production. The shift of labor necessary for the latter reduces food production and does this all the more so the lower u^*. Hence, if u^* is low, which is the case if σ is low, less labor than with a high u^* can be transferred to industry – the process lasts longer. The same argument applies to a decrease in widget consumption. This decrease could be partially compensated by substituting food for widget consumption. This is the easier, the higher u^* which implies the same effect. Just like in the previous chapter do increases in either the rate of technical progress in industry or the efficiency of research or human capital accumulation shorten the transitional period. The effect of δ seems to be stronger than that of μ, at least for the parameter combinations compiled in table 14.

Figure 13: Preference Shock with Fast Technical Progress

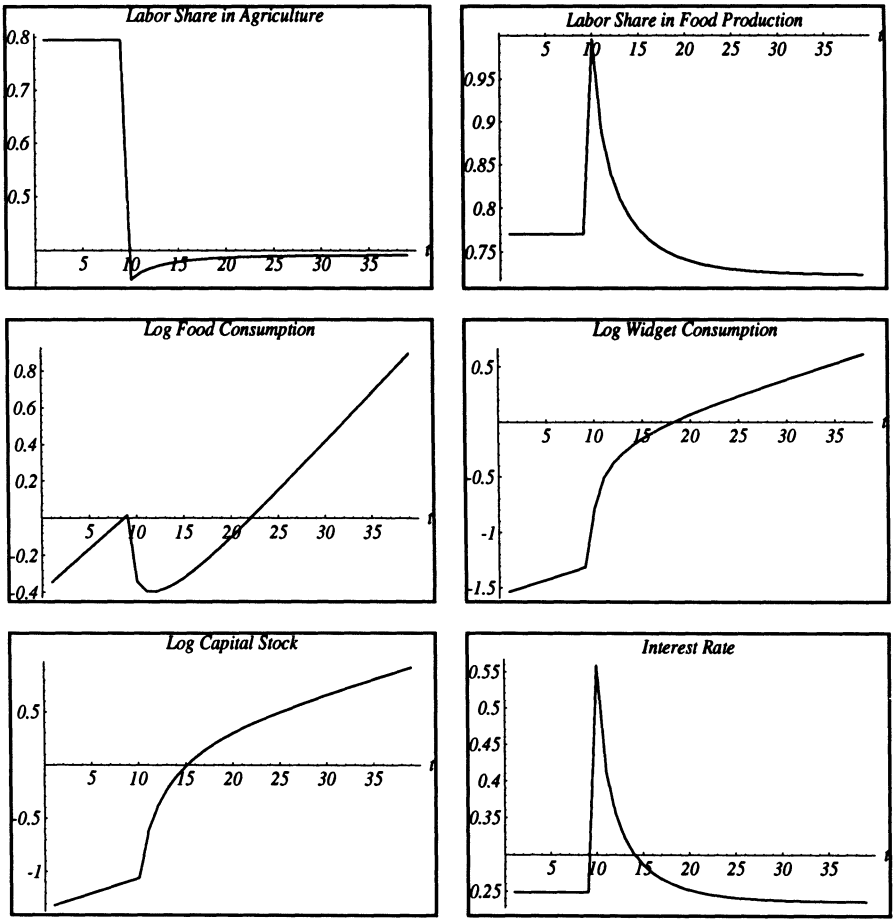
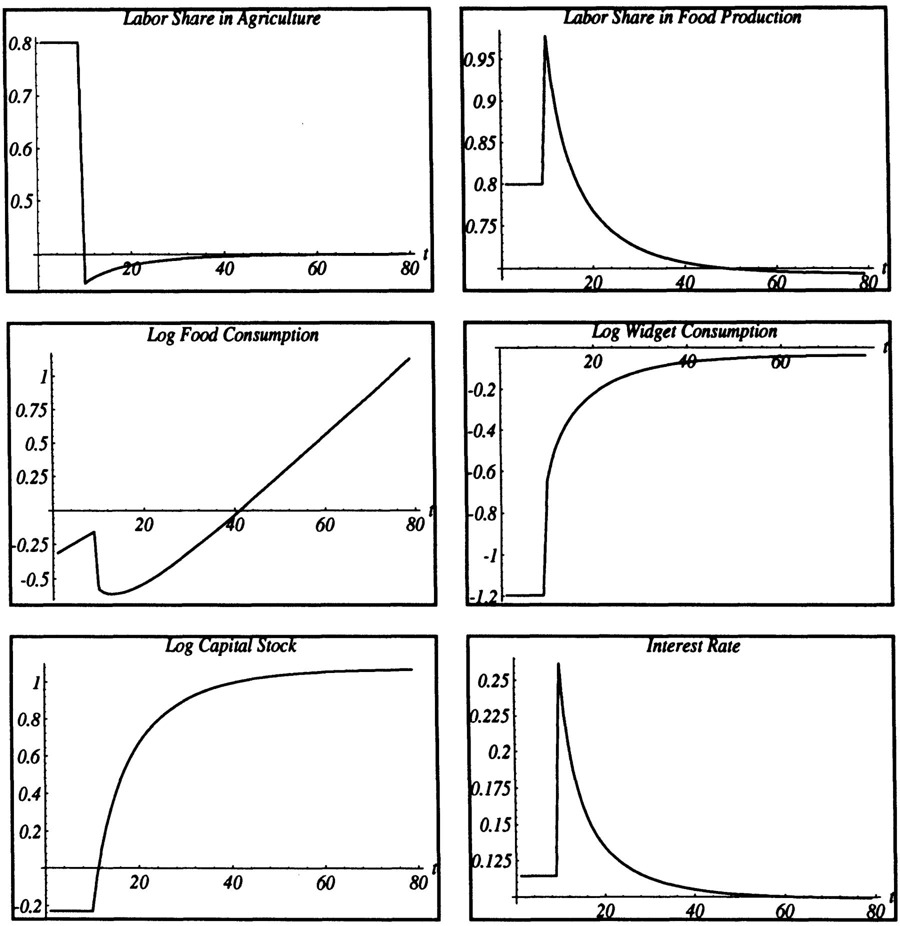


Table 14 is based on rather large shocks to the state-like variable z_2 without discussion whether these shocks are reasonable. To catch up on this question, we conduct the same experiment as in the previous chapter, namely a shock to the preference parameter γ large enough to replicate roughly the first stylized fact. We analyze two parameter constellations, one with high rates of technical progress and a second one with low rates. The first case is characterized by $\sigma = 5$, $\mu = 0.02$, $\delta = 0.2$ and the second by $\sigma = 2$, $\mu = 0$, $\delta = 0.1$.[13] As before we consider a decrease in γ from 0.8 to 0.4. The first ten periods describe the development before the preference shock. The simulation results are then depicted in figures 13 and 14.

Figure 14: Preference Shock with Slow Technical Progress



The simulations support the result from table 14 that the transition period is rather short compared to the outcome of the exogenous growth model. In the first simulation the variables have almost reached their steady-state paths after 15 years, in the second simulation after 40 years. This quick adjustment is due to the additional control variable, as the upper right panel shows: Right after the shock,

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13. The lower value for σ in the second case became necessary since the algorithm did not converge for $\sigma = 5$. This happened frequently for other parameter constellations. It is mainly caused by the boundedness of the control variables n and u. Small changes in these variables lead to large variations in the economy's time path, often leading to negative or complex values for some variable eventually which lets the numerical routines collapse.

when labor is shifted from agriculture into industry, agricultural labor is relocated from research or human capital accumulation to food production. Therefore more labor can go to industry to produce sufficient capital to reach the new steady-state growth path without suffering severe shortages in food consumption.

As in the exogenous growth model, we observe some kind of overshooting for the fraction of labor in industry. When sufficient capital is accumulated, part of the industrial labor force can migrate back into agriculture. This transition period takes longer in the situation without technical progress in industry. In this case all the additional capital needed has to be accumulated from the use of labor and capital alone. No technical progress increases the output of this sector. The simulations also show that food consumption decreases considerably for a while due to the labor shift (second panel on left). It takes more than 10 years in the first case and almost 30 years in the second case to restore the pre-shock level of food consumption.

With respect to the second stylized fact, the time-scale of industrialization, the simulations lead to the conclusion that this model can only be in accordance with the observed time-scale if preference changes happen gradually. As we have already discussed in chapter 3, this is a not an unrealistic requirement: One would assume that people learn slowly about the existence of new goods and develop new consumption habits only gradually.

While the exact results of the model are debatable since they depend on specific functional forms and parameter assumptions, some conclusions can be drawn from the simulations: first of all, industrialization is accompanied by preference changes towards consumption of the industrial output. Secondly, to produce this additional industrial output an economy has to accumulate sufficient capital. This, thirdly, requires relocation of labor towards industry, possibly initially more than will be employed in this sector in the long-run.[14] And as a fourth point, this shift of labor to industry is easier for an economy if the agricultural research or human capital accumulation is efficient. Then the loss in food production due to migration can be better compensated.

14. This points to a policy problem: the workers might be unwilling to leave and to move back to agriculture eventually. Cf. the behavior of the so-called "guest workers" in Germany.

4.4 Summary

The exogenous growth model of a dual economy derived in chapter 3 has in this chapter been extended to endogenous growth in agriculture. The equilibrium properties have remained the same: As before, the steady-state equilibrium is saddle-path stable and unique for a wide range of parameter values. This is an interesting result since previous work on two-sector models of endogenous growth has found the frequent occurrence of multiple equilibria.

Contrary to the exogenous growth model, the two sectors are not any more independent, although this result is asymmetric: While technical progress in agriculture does not influence the industrial sector – growth in the latter is still exogenous – the rate of technical progress in industry does influence agricultural growth. Expedition of industrial technical progress decreases the rate of growth in agriculture. This dependence is caused by labor shifts among sectors. Speeding up industrial technical progress rises the gains from working in this sector. It thus leads to migration from agriculture into industry. The reduction of labor in food production now raises the marginal utility from working in production instead of in research or human capital accumulation. The resulting migration is responsible for slower growth.

The quantitative properties of the endogenous growth model are quite similar to the results obtained above: Changes in the industrial rate of technical progress and the efficiency of research or human capital accumulation in agriculture have only small effects on the economy's structure. As before, a change in preferences is the only possibility to explain the observed migration of labor from agriculture into industry. However, the transitional dynamics are now much shorter. This is due to the endogenization of agricultural technical progress. Adjustments in this activity enable a country to reach its steady-state much faster. Endogenizing further decisions, for example the rate of technical progress in industry, would shorten this transition period further.

Due to the shorter transition period structural change now happens considerably faster than the stylized facts suggest. This outcome seems to discredit the model at first sight. But in reality research or human capital accumulation decisions *are* to a large extend endogenous. Even many other decisions, which are taken as exogenous in this model, are the result of deliberate actions by the individuals. This makes the real economy even more flexible than the model presented here. One should also keep in mind that the model here is an *optimal control* model assuming that the economy is controlled to stay on its optimal path. It should therefore better serve as a benchmark case describing the best possible paths for growth and development. That structural change took a larger time in reality, might simply highlight the fact that real economies are not always on their optimal paths. However, the gap to reality could also have been caused by a more gradual preference change than assumed here.

One possible reason for non-optimality could be externalities in agricultural research or human capital accumulation as the model has shown. Such an economy would exhibit lower rates of growth in agriculture than optimal. Contrary to intuition, however, it would have a larger fraction of labor in industry than optimal. This outcome is caused by the agents' underestimation of the gains from staying in agriculture and investing in research or human capital accumulation instead of migrating to industry. Economic policies to increase the rate of growth in agriculture are therefore also a possibility to keep the population in agriculture or, equivalently, to prevent over-urbanization. Such a policy could be, for example, governmental research to overcome the externality problems from agricultural production techniques. An equivalent is compulsory schooling to raise the level of education beyond that which individuals would acquire on their owns.

To conclude, the assessment of this chapter's dual economy model is contradictory. On the one hand it turned out to be useful to study the macroeconomic effects from microeconomic decisions to increase productivity. The existence of externalities in this context can help to explain some common problems in developing countries like too low productivity growth rates in agriculture or over-urbanization. On the other hand the model is only partially able to replicate major stylized facts of economic development. This is partly due to the omission of sluggish mechanisms like Engel's law. The latter will be discussed in chapter 6.

5. Technology Adoption and Catch-Up

So far technology in agriculture has either been assumed as growing exogenously (chapter 3) or as being immediately adopted after invention (chapter 4). While this has provided some insights about growth and structural change it has totally neglected the process of technology adoption itself which can be defined as "the process of spread of a new technology within a region" (*Feder, Just,* and *Zilberman,* 1984, 257). In this chapter the models derived previously are modified to discuss technology adoption in agriculture. The questions we want to answer are: How does an economy's growth behavior and its structure change if technology is adopted from some technologically superior country instead of being created from scratch? Does convergence, a catch-up to income level and growth rate of this leading country take place? If yes, what determines extend and speed of this convergence? What effects do economic policies to increase technology adoption have? As before, we focus only on the agricultural sector, although similar questions can be asked about industry.

We will proceed in three steps: in the first section we present some peculiarities of technology adoption in agriculture and compare these insights with commonly made assumptions about this process. In the next section a model based on chapter 3 is presented where the rate of technology adoption is exogenously given; this will give a basic idea of the dynamics. The third section then discusses a model where agents decide endogenously about how much technology to adopt when this process is costly. This model is based on chapter 4.

5.1 Adoption of Technologies in Agriculture

The topic of technology adoption in the agricultural sector has always been one of the central themes in development economics. Numerous empirical and theoretical studies have discussed the process of and obstacles to technology adoption in agriculture. Recently, another strand of – mainly empirical – literature has evolved which does not discuss technology adoption itself but rather builds on a specific assumption about international technology adoption. The central hypothesis of this so-called "convergence discussion" is that easy transferability of technology from industrialized to developing countries should lead to a rapid "catch-up" of these late-comers.

This assumption of easy technology transferability from industrialized to developing countries resembles one of the central ideas in the early development literature from the 1950s and 1960s as we have already mentioned in section 2.4.

> In the early days of development efforts, it was fashionable to regard the availability of all of the blueprints developed over the past several hundred years as a major source of advantage to developing countries. Some went so far as to regard 'technology transfer' as the essence of development. (*Krueger*, 1991, 463f.)

This pool of technologies was seen as a major advantage for developing countries and formed the basis for the myth (UNCTAD, 1990, 22) that these countries had the possibility of technological leapfrogging.

Consequently, the common development policy was to set up "extension services" which had the task of screening all available technologies and extending suitable ones to farmers. However, it became soon clear that "agricultural technology is simply not very transferable." (*Judd, Boyce, Evenson*, 1986, 78) Technologies – especially for agricultural production – are usually developed in response to a country's relative factor endowments, the nutrition habits of its inhabitants, its soil, and climate. Especially the latter "natural" conditions in the developing South are totally different from those in the industrialized North. But even with similar climatic conditions the technologies in use are often different. *Hayami* and *Ruttan* (1985, part III), for example, show that in response to its relative factor endowments the United States and Japan developed different agricultural techniques: labor saving by mechanization in the US and land saving by chemical inventions in Japan. The development strategy of the 1950s neglected the necessity for technologies to correspond to factor endowments and other conditions. It turned out to be not very successful: "The lesson of the past three decades or so is that the availability of technology per se is seldom of much use... When technologies are inappropriate, the net benefits may even be negative." (*Krueger*, 1991, 464)

While soon dismissed in the development literature, the idea of free floating technological knowledge turned up again in growth theory. *Romer* (1993, 546) claims that "... people in the industrial nations of the world already possess the knowledge needed to provide a decent standard of living for everyone on Earth." This belief is also the major assumption behind the so-called "convergence" discussion. This discussion is based on assumptions about technological knowledge in general without distinguishing between agriculture and industry which might be one reason for neglecting the evidence about technology transferability from the development literature. It evolved in the late 1980s from the revival of growth theory and interpreted the simple neoclassical growth model as implying convergence in growth rates of per-capita income across countries as *Solow* (1991, 5) points out:[1]

> New technology is accessible anywhere in the world. Knowledge circulates easily and quickly. The model should therefore imply that the steady-state growth rate would be the same in all countries. Allowing for non-steady-state behavior leads to an even stronger implication: poorer countries should grow faster than richer countries and national growth rates should diminish over time. The world thus converges to a steady state in which every country has the same per capita growth rate.

With slightly stronger assumptions, free technology transferability even implies converge to similar per-capita income *levels*.[2] This convergence hypothesis has produced a tremendous amount of empirical work (surveyed, for example, by *Levine* and *Renelt*, 1992; *Barro* and *Sala-i-Martin*, 1992; *Sala-i-Martin*, 1994), although much of this work has been criticized as being too simplistic. One of the first critiques was formulated by *Mankiw*, *Romer*, and *Weil* (1992), who have pointed out that one cannot investigate convergence with the simple *Solow* model since not all countries converge to the same steady-state. They augment the model with human capital accumulation and get results in favor of the modified convergence hypothesis. Lately, however, the whole convergence literature has been questioned on methodological grounds (*Friedman*, 1992; *Quah*, 1993; an overview is given by *Bernard* and *Durlauf*, 1994).

In the light of the developing countries' experience with technology transferability the convergence discussion has to be questioned even more. Its central hypothesis, free transferability of technology, is simply not very plausible. This poses the ques-

1. An early example of this argument is *Romer* (1986). He interpreted the missing evidence in favor of the simple convergence hypothesis as evidence against the orthodox-neoclassical growth model while supporting his own, endogenous growth model with increasing returns to scale.
2. This is the case if preferences, production techniques, and other factors like, e.g., labor growth rates are identical and multiple equilibria can be ruled out.

tion what, if anything, remains from the convergence hypothesis if new technologies do not enter production automatically but have to be adopted – possibly costly. This is one of the questions posed at the outset.

Some guidance for answering the questions is offered from the first literature strand mentioned above, namely from discussions of the technology adoption process itself. First of all, to clarify the process of technology adoption, it is useful to distinguish between individual (farm level) adoption and aggregate adoption.

> Final adoption at the level of the individual farmer is defined as the degree of use of a new technology in long-run equilibrium when the farmer has full information about the new technology and its potential. [...] Aggregate adoption is measured by the aggregate level of use of a specific new technology within a given geographical area or a given population. (*Feder, Just, Zilberman,* 1984, 256f.)

Both adoption processes have been studied extensively in theoretical as well as empirical work.[3] For the *individual* farmer's decision to adopt a new technology, several factors are of importance: the farm size, her behavior towards risk and uncertainty, her endowment with human capital, the availability of labor, credit constraints, as well as tenure arrangements between land owners and farmers. While most of these influences cannot be studied easily within the models derived in the previous chapters, the effects of education can. In addition, human capital has turned out in many empirical studies to be an important determinant of technology adoption.[4] The empirical evidence suggests that better educated farmers are earlier adopters of modern technologies and apply modern inputs more efficiently during the adoption process (*Feder, Just, Zilberman,* 1984, 276).

In addition to the individual farmer's behavior the *aggregate* adoption process includes the behavior of institutions that make knowledge available to the individual farmers in a country or region, or of institutions that adapt technologies to local conditions. These institutions can be extension services, farmer co-operatives, national research organizations, or even international agricultural research

3. Cf. the survey by Feder, Just, Zilberman (1984).
4. *Nelson* and *Phelps* (1966) point out that human capital is especially important in agriculture since each farmer has to decide herself about adopting a new technology and has to conduct this adoption. In industry this decision is mostly made by few highly specialized managers or researchers while the majority of workers only has to adapt to the new technology.

centers (IARCs). *Abramowitz* (1986) goes even further in stating that a country's ability to take advantage of a technological gap and to put more production technologies in use is a function of its "social capability". This capability includes education and also several other aspects of the economic system like openness to competition or to the establishment and operation of new firms. The importance of institutional factors in a broad sense is also emphasized by *Parente* and *Prescott* (1992, 1994). They point out that government policies as well as the actions of individuals or groups of individuals other than those making the adoption decisions substantially affect the return to technology adoption.

The institutions have three options to handle the stream of knowledge and agricultural technologies (*Evenson* and *Binswanger*, 1978), which are to a considerable extend being produced in industrialized countries:[5] First of all, they can screen the available foreign techniques and adopt the most suitable one without adaptation (direct transfer). Secondly, they can select some foreign techniques which can subsequently been modified through adaptive research to suit local conditions. Finally, the third option involves screening of technology and basic scientific knowledge as foundation for own comprehensive local research. The output from these institutions will then depend on their research and adaptation efficiency which, among other things, depends on their endowment with human capital as well as on the transferability of their input technologies.

No matter whether individual or aggregate technology adoption is considered, in both cases it does not take place without effort. In both cases the agents incur costs: in the individual case a farmer can adopt a new technology faster if she is better educated. But to accumulate a sufficient amount of human capital she has to visit school instead of working in food production. Also the adoption process itself is costly. The farmer has to spend time, which could otherwise been used in actual production, to gather information about new technologies, to understand

5. While agricultural research in developing countries has increased, the industrialized countries of the world – Western Europe, Eastern Europe/Soviet Union, and North America/Oceania in 1980 still had the largest share in world agricultural research expenditure, roughly 65%. Taking scientist man-years as measure since scientists are considerably cheaper in developing countries, reduces this share to slightly less than 50%. However, most of the remaining share accrues to Asia, especially China. Latin America and Africa only have a share of roughly 5% in either category (*Judd*, *Boyce*, and *Evenson* 1986).

them, and to evaluate their profitability. In aggregate adoption at least a part of the technology available from technological leaders has to be adapted. This is only possible if the population devotes resources to research in order to perform this adaptation.

For the remainder of this chapter we will always identify adoption with aggregate adoption unless stated otherwise, while keeping in mind that aggregate adoption also includes individual adoption activities. We will assume that the developing country is a technological follower and call it simply "the South". There exists a technological leader, "the North", which is not specified further. The developing country does not engage in own, independent agricultural research but rather adapts the North's technology to its own needs.[6] Within this framework we will try to answer the questions posed at the outset of this chapter. As before, we will discuss the basic issues algebraically and resort to numerical simulations for the remainder, especially for discussing the transitional dynamics.

5.2 Exogenous Technology Adoption

In chapter 3 a model with exogenous technical progress in agriculture has been presented. This model can easily be extended to technology adoption. We assume that the South adopts the agricultural technology created in the North with a constant exogenous rate.[7] Technology adoption is not costly since no resources have to be devoted to this activity. However, new technologies are not immediately productive, either. We will first discuss the consequences of this different set-up for the exogenous growth model and afterwards discuss the new model's implications for convergence. At the end of this section we will again conduct a policy experiment by means of numerical simulation.

Suppose that the total number of available agricultural technologies or blueprints produced in the North, which is denoted by A, grows with a constant exogenous rate ν. To increase agricultural productivity, these technologies have to be adapted by some institutions to the local conditions and finally have to be adopted by

6. This rules out any discussion of overtaking in technology levels.

7. This simple set-up is an adaptation of *Helpman*'s (1993) model for analyzing patent protection in developing countries.

farmers. Denote the *number* of already adopted technologies by A_A and the number of remaining technologies by A_N with $A_A + A_N = A$. The adoption happens with a constant rate $m = \dot{A}_A / A_N$. With this modification the agricultural production function from chapter 3 changes into (we assume again for simplicity that labor is constant and normalized to unity):

$$(5.1)\quad Y_A = A_A n^{\alpha}, \qquad 0 < \alpha < 1$$

which can also be written as $Y_A = aAn^{\alpha}$ where $a = A_A / A$ denotes the *fraction* of technologies already adopted.

Since $\dot{a}/a = \dot{A}_A/A_A - \dot{A}/A$, the fraction of converted technologies obeys the differential equation[8]

$$(5.2)\quad \dot{a} = m - (\nu + m)a.$$

In steady-state equilibrium this fraction a has to be constant since m and ν are constant. Its steady-state value can thus be derived from equation (5.2) as:

$$(5.3)\quad a^* = \frac{m}{\nu + m}$$

The steady-state value of adopted technologies will only be unity and the economy will only have adopted all available technologies eventually if the North is technologically stagnant ($\nu = 0$) or the rate of adoption is infinitely large.

Since a is constant in the steady-state, the long-run growth rate of technology in agriculture and therefore also the long-run growth rate of per-capita food consumption due to $\dot{A}_A/A_A = \dot{a}/a + \dot{A}/A$ reduces to

$$(5.4)\quad \left(\frac{\dot{c}_A}{c_A}\right)^* = \left(\frac{\dot{A}_A}{A_A}\right) = \nu.$$

This outcome is the same as in chapter 3 with $\lambda = 0$ apart from the small but important difference that ν is now determined in the North. The remaining steady-state values are thus similar to chapter 3:

8. $\frac{\dot{A}_A}{A_A} = \frac{\dot{A}_A}{A_N}\left(\frac{A - A_A}{A_A}\right) = \frac{\dot{A}_A}{A_N}\frac{1}{a} - \frac{\dot{A}_A}{A_N} = \frac{m}{a} - m.$

$$\left(\frac{\dot{c}_M}{c_M}\right)^* = \left(\frac{\dot{K}}{K}\right)^* = \frac{\mu}{\alpha}$$

$$n^* = \frac{\gamma\left(\rho - (1-\sigma)\left(\gamma\nu + (1-\gamma)\frac{\mu}{\alpha}\right) + \mu\right)}{\rho - (1-\sigma)\gamma\nu + \sigma(1-\gamma)\frac{\mu}{\alpha} + \gamma\mu} \tag{5.5}$$

While the growth rates of capital and widget consumption are still determined in the South, the economy's structure is influenced by research activity in the North.

This model is rather pessimistic in implying that economic policy aiming at improving the adoption process by increasing the efficiency of the adapting institutions or by promoting adoption of new technologies on the individual farm level neither has long-run growth effects nor influences the country's degree of industrialization. This implication is similar to the growth pessimism resulting from *Solow's* (1956) simple neoclassical growth model which states that economic policy can only have level effects, not growth effects, except by enhancing technical progress.[9]

Economic policy does have level effects by influencing the steady-state fraction of adopted technologies, a. From equation (5.3) we can see that any increase in the rate of adoption m raises the steady-state value of a (i.e., makes the gap to the technological leader smaller) and thus shifts the growth path of food consumption higher.

Equation (5.3) shows that there will never be full convergence in levels, except in very special cases. Thus, the hypothesis of "strong convergence" in levels, as stated by part of the convergence literature, cannot be supported by this model. We cannot make any statements, though, about GDP growth rates. This would require assumptions about the development of relative prices which is beyond the scope of the analysis conducted here. However, the model shows that a convergence in agricultural growth rates takes place: in the long-run does food consumption in the South grow with the same rate as in the North. While this seems to support

9. Part of this pessimism stems from the assumption of a constant labor force. With $\lambda > 0$ it is easy to see that the growth rate of food consumption depends negatively on λ just like in chapter 3. Decreasing this rate would then be an appropriate policy to rise the growth rate of per-capita food consumption.

the convergence hypothesis, the model points to a flaw in it: it shows that in the steady-state the *growth rates* of per-capita food consumption are equal while the *levels* remain permanently different. Thus, lower levels of food consumption do not automatically imply subsequently higher growth rates which refutes the central hypothesis being tested in most of the empirical convergence literature.

Consider now the short-run dynamics of the equation system. One can think of shocks to the system initiating a transitional period as being caused by different events. For example, any policy that lets the developing country participate more in the international flow of ideas, increases A, the number of available technologies by adding new technologies to the pool of not yet converted A_N. Such policies might be exchange programs for agricultural researchers or a less stringent patent enforcement on part of the North. Hence, since $a = A_A / A$, such an event would constitute a negative shock to a which subsequently increases again to its steady-state value. While a returns to its steady-state value eventually, such a policy of increasing A raises the growth path of A_A in the long-run. A second possible policy would be the establishment or improvement of agricultural research centers in the South. In this model it would be interpreted as an increase in m leading to a new, higher steady-state level of a. The consequence is also a level effect and the transitional dynamics are the same as those following the first policy. If m is increased, a is in the beginning below its new steady-state value.

Similar to the previous chapters, we can simulate the transitional dynamics numerically. This yields some estimates about the length of this process and about the influence of technology adoption on other variables in the model. The effect we want to consider is an increase in the rate of technology adoption. This economic policy one would intuitively advocate for technological followers to catch up with a leader. By equation (5.3) this policy raises the steady-state fraction of adopted technologies in the pace given by (5.2).

Technically, the simulation is conducted in the same way as in chapter 3. First of all, it is easy to see that the model's dynamics are again determined by equations (3.19) and (3.20), since technology adoption happens exogenously and therefore does not change the individuals' behavior. The only modification is that ν in equation (3.20) is now substituted by

$$\frac{\dot{A}_A}{A_A} = \frac{\dot{a}}{a} + \nu = m\frac{(1-a)}{a}.$$

With a evolving according to equation (5.2) and $\lambda = 0$ the system of differential equations describing the economy's motion over time, which was given by equations (3.19) and (3.20), becomes now:

$$\dot{a} = m - (\nu + m)a$$

$$\dot{z}_2 = z_2\left(\mu - \alpha z_2 (1-n)^{\alpha - 1}\left(\frac{\gamma - n}{\gamma}\right)\right)$$

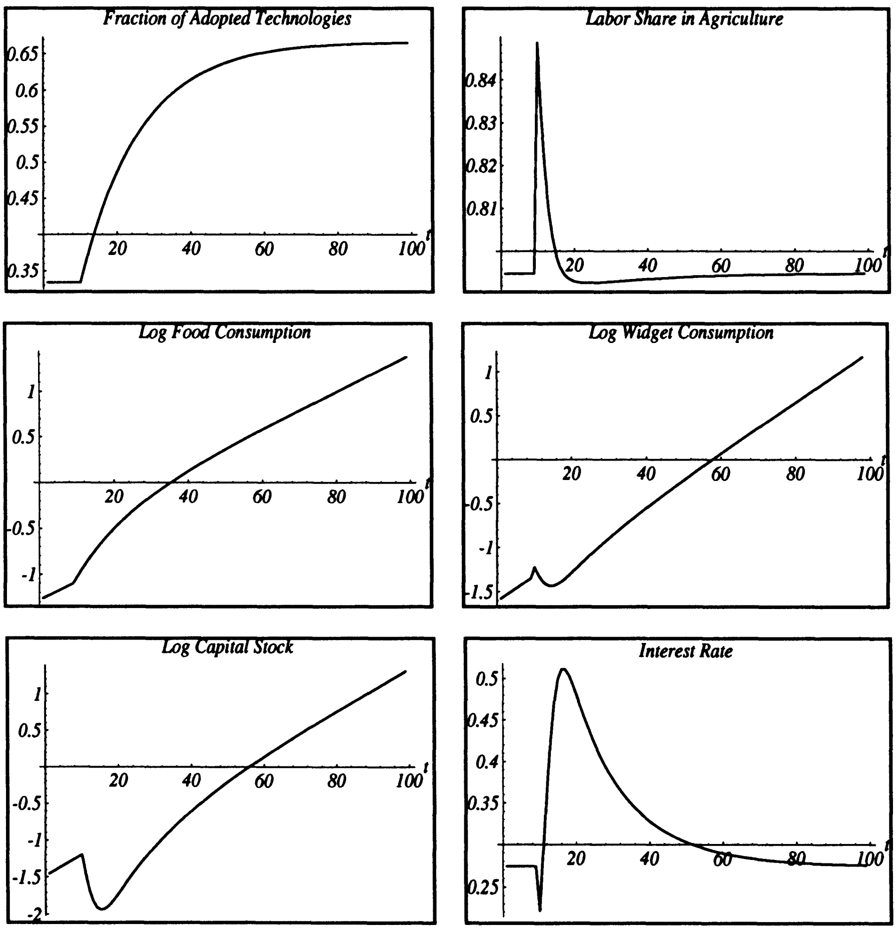
(5.6)
$$\dot{n} = \frac{n(1-n)\left[(1-\sigma)\left(\mu(1-\gamma) + \gamma m\frac{(1-a)}{a}\right) - \mu - \rho\right]}{\sigma(1-\alpha n) + (1-\sigma)\gamma(1-\alpha)} + \frac{(1-\alpha)(1-\gamma)z_2 n(1-n)^{\alpha}\frac{(\sigma n + \gamma(1-\sigma))}{\gamma}}{\sigma(1-\alpha n) + (1-\sigma)\gamma(1-\alpha)}$$

This is a system in three variables: control n, state-like variable z_2, and exogenous state-variable a. Since there are now two state-variables, we can neither calculate a single policy function depending on z_2 as we did in chapter 3 ($\dot{a}$ depends only on a and time) nor can we calculate two control equations as in chapter 4 since there is only one control but two state variables. We therefore modify the time-elimination method by choosing a two-step approach: In the first step, $n'(z_2)$ is calculated exactly like in chapter 3 with a taking on its first-period value. Then the value of a for the next period is calculated according to the first equation of (5.6). Next, we calculate $n'(z_2)$ with the new a. This procedure is repeated until a has reached its steady-state. The intuition behind this procedure is the following: in every period the stable trajectory conditional on a is calculated. For each a exists a different trajectory. The combination of movements on this trajectory (for n and z_2) and movements between trajectories (due to changes in a) describes the evolvement of the economy's variables over time.

With this algorithm we have analyzed the effects of an increase in m from 0.1 to 0.4. These values correspond to doubling the steady-state value of a^* from 1/3 to 2/3. The values are not too far from reality. *Hayami* and *Ruttan* (1985, 123) have calculated values for land and labor productivity for a variety of countries. Average agricultural labor productivity in less developed countries in 1980 was about 6%

of that in developed countries while land productivity (output per hectare) was around 49%. Our starting value is between those estimates.[10] The remaining parameters are chosen as $\rho = 0.05$, $\alpha = 0.7$, $\gamma = 0.8$, $\mu = \nu = 0.02$, $\sigma = 10$. The results are plotted in figure 15.[11]

Figure 15: Increasing the Rate of Technology Adoption



These graphs first of all show that transitional dynamics last for a rather long time. Only after about 90 years is a close to its new steady-state value. The effects

10. Note that these values are not directly applicable. In our model technical progress is neutral while direct transfer of technologies from the North is usually factor-biased. Cf. footnote 5 in chapter 4.

11. Different parameterizations yield similar pictures.

of this transition on the remaining variables, though, are small. The fraction of labor in agriculture, n, returns rather quickly to a value close to its steady-state. Also the peak in its time-path is rather small. Similarly, although slightly slower, does the capital stock return to its steady-state growth path. This rather long transitional period is due to the exogeneity of technology adoption.

The spikes in capital growth and the real interest rate are both caused by the short-run labor shift into agriculture. The reason for this shift can be seen from the labor market condition (3.7) which for this section changes into

$$\frac{\partial H_c}{\partial n} = \alpha\gamma\left[(Aan^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} n^{-1} - \alpha\theta M K^{1-\alpha}(1-n)^{\alpha-1} = 0.$$

The increase in m, which means that a is below its new steady-state value in the beginning, increases the marginal utility from working in food production. Due to the low a less food is produced in the steady-state than optimal. Since the increase of a due to technology adoption requires time, labor is shifted into agriculture for a while until a has risen. During this time production in industry is less than optimal which leads to a dent in capital accumulation. To catch up again, subsequently a larger labor share than in the long-run is employed in industry until the economy has reached its new steady-state.

The level effects derived analytically for food consumption can be seen clearly. They are, however, not too large. This outcome shows again the small impact of growth versus level effects. While the (strong) policy increases food consumption by roughly 80% in 50 years, the basic effect of technical progress of 2% a year over the same time increases food consumption by 170%. It therefore seems that there is not too much to get from such a policy which looked rather promising at first sight. On the other hand, since technology adoption is costless in this simple model, it does not harm either.

5.3 Endogenous Adoption Decisions

In this section the technology adoption decision is endogenized. This is achieved by modifying the model from chapter 4. Contrary to previous section 5.2 the process of technology adoption now becomes costly.[12] We assume that the rate of technology adoption in the agricultural sector is a function of the resources devoted to this activity. The activity can either be adaptive agricultural research as outlined above in section 5.1 or human capital accumulation. As before, agricultural technology is assumed to originate in the technologically leading North. The basic idea behind this model has been developed by *Nelson* and *Phelps* (1966) in the context of a simple neoclassical growth model and has recently been subject to empirical tests by *Benhabib* and *Spiegel* (1994).[13]

Devoting resources to technology adoption encompasses all aspects of aggregate adoption mentioned in the introduction to this chapter. It includes allocation of resources to agricultural research centers for conducting adaptive research as well as the individual farmer's allocation of time to understanding, modifying, and eventually implementing new technologies. The central point is that labor, which could otherwise be used in production, is employed in adoption of new technologies which increase future productivity. This trade-off also exists for another possible interpretation familiar from chapter 4: The rate of technology adoption depends also on the level of human capital in agriculture. Obtaining and keeping a certain level of human capital requires allocation of time to schooling, time that could otherwise be used for producing food. Indeed, this positive effect of education is what much of the empirical work on technology adoption concentrates on as we have mentioned above.

Suppose that individuals decide endogenously about how much time to spend in technology adoption. We assume that the rate of technology adoption is chosen according to the following equation:

$$\text{(5.7)} \quad \frac{\dot{A}_A}{A_N} = \delta(1-u)$$

with A_A denoting the number of already adopted technologies and A_N the remaining not (yet) adopted technologies. The total number of technologies $A = A_A + A_N$ grows with a constant exogenous rate ν. The term $(1-u)$ denotes

12. *Easterly, King, Levine,* and *Rebelo* (1994) also present a model where technology adoption is costly. They simply assume that adoption costs are proportional to the size of the labor force since a larger labor force requires that more workers become familiar with the new technique.

13. See also footnote 6. in chapter 4.

the fraction of agricultural labor engaged in technology adoption and the parameter δ the efficiency of this process. Equation (5.7) is very similar to the assumption about exogenous technology adoption made in the previous section. Here the exogenous parameter m has been replaced with the endogenous term $\delta(1-u)$. The equation can be rewritten in the more familiar form giving the growth rate of adopted technologies in agriculture as

$$\frac{\dot{A}_A}{A_A} = \delta(1-u)\left(\frac{A-A_A}{A_A}\right).$$

With this new equation determining the rate of growth of adopted technologies in agriculture, the problem (4.3) from chapter 4, which a social planner has to solve, changes into:

$$(5.8)\qquad \begin{aligned} &\max_{c_M,\, n,\, u} && \int_0^{\infty} \frac{\left[\left(A_A^{\eta}(un)^{\alpha}\right)^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} e^{-\rho t}\,\mathrm{dt} \\ &\text{s.t.} && \dot{K} = MK^{1-\alpha}(1-n)^{\alpha} - c_M \\ &\text{and} && \dot{A}_A = A_A\delta(1-u)\left(\frac{A-A_A}{A_A}\right) \end{aligned}$$

The new current-value Hamiltonian for the optimal solution[14] is given by:

$$(5.9)\qquad H_c = \frac{\left[\left(A_A^{\eta}(un)^{\alpha}\right)^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} + \theta_1\left(MK^{1-\alpha}(1-n)^{\alpha} - c_M\right) + \theta_2\delta A_A(1-u)\frac{(A-A_A)}{A_A}$$

which leads again to seven solution equations:

$$(5.10)\qquad \frac{\partial H_c}{\partial c_M} = (1-\gamma)\left[\left(A_A^{\eta}(un)^{\alpha}\right)^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} c_M^{-1} - \theta_1 = 0$$

$$(5.11)\qquad \frac{\partial H_c}{\partial n} = \gamma\alpha\left[\left(A_A^{\eta}(un)^{\alpha}\right)^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} n^{-1} - \alpha\theta_1 MK^{1-\alpha}(1-n)^{\alpha-1} = 0$$

14. We only discuss the optimal solution here since the difference between optimal and market solution is equivalent to that already discussed in chapter 4.

$$(5.12)\quad \frac{\partial H_c}{\partial u} = \gamma\alpha\left[(A_A^{\eta}(un)^{\alpha})^{\gamma}c_M^{1-\gamma}\right]^{1-\sigma}u^{-1} - \theta_2 A_A\delta\left(\frac{A-A_A}{A_A}\right) = 0$$

$$(5.13)\quad \frac{\partial H_c}{\partial \theta_1} = \dot{K} = MK^{1-\alpha}((1-n))^{\alpha} - c_M$$

$$(5.14)\quad \frac{\partial H_c}{\partial \theta_2} = \dot{A}_A = A_A\delta(1-u)\left(\frac{A-A_A}{A_A}\right)$$

$$(5.15)\quad \dot{\theta}_1 = \theta_1\rho - \frac{\partial H_c}{\partial K} = \theta_1\rho - \theta_1(1-\alpha)MK^{-\alpha}(1-n)^{\alpha}$$

$$(5.16)\quad \dot{\theta}_2 = \theta_2\rho - \frac{\partial H_c}{\partial A} = \theta_2\rho + \theta_2\delta(1-u) - \gamma\eta\left[(A^{\eta}(un)^{\alpha})^{\gamma}c_M^{1-\gamma}\right]^{1-\sigma}A^{-1}$$

Equations (5.10) - (5.16) together with the initial values A_{A0} and K_0 as well as the transversality conditions $\lim_{t\to\infty} e^{-\rho t}\theta_1 K = 0$ and $\lim_{t\to\infty} e^{-\rho t}\theta_2 A_A = 0$ describe the economy's motion through time.

Consider again the steady-state equilibrium first. The growth rates of capital and per-capita consumption of manufacturing goods are obtained in the same way as in chapter 4 as:

$$(5.17)\quad \left(\frac{\dot{c}_M}{c_M}\right)^* = \left(\frac{\dot{K}}{K}\right)^* = \frac{\mu}{\alpha}.$$

The steady-state growth rate of adopted technologies can be obtained from equation (5.14). Since u must be constant in the steady-state, this growth rate can only be constant if $(A - A_A) / A_A = A / A_A - 1$ does not change either. This requires that A and A_A grow with the same rate. Thus,

$$(5.18)\quad \left(\frac{\dot{A}_A}{A_A}\right)^* = \nu.$$

The steady-state rate of technical progress in the South's agricultural production is not any more endogenous but rather determined by innovation activity in the North. This is the same result as with exogenous technology adoption.

These growth rates are sufficient to derive the parameter restrictions for ρ derived from the two transversality conditions. This is done in appendix A.12 and yields the requirement that

$$\text{(5.19)}\quad \rho > (1-\sigma)\left(\gamma\eta\nu + (1-\gamma)\frac{\mu}{\alpha}\right).$$

The results derived so far are also sufficient to derive the steady-state structure of the economy which is given by the fraction of labor in agriculture, n^*. This is done in appendix A.13 and leads to

$$\text{(5.20)}\quad n^* = \frac{\gamma\left(\rho - (1-\sigma)\left(\gamma\eta\nu + (1-\gamma)\frac{\mu}{\alpha}\right) + \mu\right)}{\rho - (1-\sigma)\gamma\eta\nu + \sigma(1-\gamma)\frac{\mu}{\alpha} + \gamma\mu}.$$

The steady-state fraction of labor in agriculture, n^*, from equation (5.20) equals that from previous section. The growth rate of agricultural technology in the North, ν, does influence the economy's structure. Contrary to the endogenous growth model in section 4, the parameter δ, which in this chapter denotes adoption efficiency, does not influence n^* anymore. Therefore an economic policy aiming at increasing this factor, for example by increasing the efficiency of schooling, does not anymore lead to a less industrialized economy.

The difference to the model of exogenous technology adoption is the choice of u^*, the fraction of agricultural labor in actual production. Combination of equations (5.12), (5.16), and differentiation of (5.12) yields

$$(1-\sigma)\left(\gamma\eta\nu + (1-\gamma)\frac{\mu}{\alpha}\right) = \rho + \delta(1-u) - \frac{\eta\delta u}{\alpha}\left(\frac{A - A_A}{A_A}\right) + \nu.$$

The levels of A and A_A can be eliminated by substituting (5.14) into this equation while making use of equation (5.18). This leads to a quadratic equation in u with the following solution:

$$\text{(5.21)}\quad u^* = 1 + \frac{B + \frac{\eta\nu}{\alpha}}{2\delta} \pm \sqrt{\frac{\left(B + \frac{\eta\nu}{\alpha} + 2\delta\right)^2}{4\delta^2} - \frac{B}{\delta} - 1}$$

where $B = \rho + \nu - (1-\sigma)\left(\gamma\eta\nu + (1-\gamma)\frac{\mu}{\alpha}\right).$

Only one of these solutions is feasible. Since $B > 0$ by transversality condition (5.19), a necessary condition for $u^* < 1$ is that the square root must be subtracted from the first term. Thus the second solution can be ruled out. We show in appendix A.14 that also $u^* > 0$.

Last, from equations (5.21), (5.18), and (5.14) we can derive the endogenous steady-state values for $a = A_A / A$, the fraction of adopted technologies. By equations (5.14) and (5.18)

$$\nu = \delta(1-u)\left(\frac{A-A_A}{A_A}\right) = \delta(1-u)\left(\frac{1-a}{a}\right)$$

and thus

$$\text{(5.22)} \quad a^* = \frac{\delta(1-u^*)}{\nu+\delta(1-u^*)}.$$

This outcome is again similar to the previous section with $\delta(1-u^*)$ in the endogenous adoption model corresponding to m in the exogenous adoption version. Similar to the latter there will never be full convergence in levels as equation (5.22) shows, except when $\delta \to \infty$ or $\nu = 0$. Even if the South used all agricultural labor for adopting technologies ($u^* \to 0$), it would not reach the technology and therefore productivity level of the North. Thus, even the endogenous technology adoption model does not support the central idea behind the convergence hypothesis that lower levels of productivity imply subsequently higher growth rates.

Consider now the effects of economic policy. A roughly equivalent policy to an increase in m, the rate of technology adoption from last section, would be an increase in δ, the adoption efficiency. Such a policy could be an improvement in agricultural research centers or a better support to farmers who want to adopt new technologies, for example, by making a larger amount of information available to them. According to equation (5.21) such an increase in δ would raise u^* as is shown in appendix A.15. This is just the opposite effect as in the endogenous growth model of chapter 4.

Equation (5.12) tells something about the dynamics behind this effect: increasing δ raises the marginal utility from engaging in research, implying a shift of labor into this activity. Hence, the growth rate of A_A rises. However, contrary to the model in chapter 4 this is only a short-run phenomenon, as equation (5.14) shows since in the long-run the growth rate of adopted technologies is given by ν. Therefore labor moves back into production eventually.

In the long-run the engagement of research or human capital accumulation is governed by equation (5.14) and the exogenous ν. Nevertheless, an increase in δ induces a rise in a^*, as is also shown in appendix A.15. Thus, the level of c_A rises by two effects through such a policy, by higher u^* and by increased a^*. However, it never reaches the North's level since there is never full adoption. If costs of technology adoption are decreased (by increasing δ), the South chooses to spend less time in technology adoption, not more, because more adoption has only level effects.

Simulation of the transition dynamics is rather difficult with this model since there are now two state-variables (z_2, a) and two controls (n, u). Contrary to last section, the modified time-elimination method cannot be used anymore. Movements on and between trajectories are not any more independent since a is now chosen endogenously. The growth rate of a is given by:

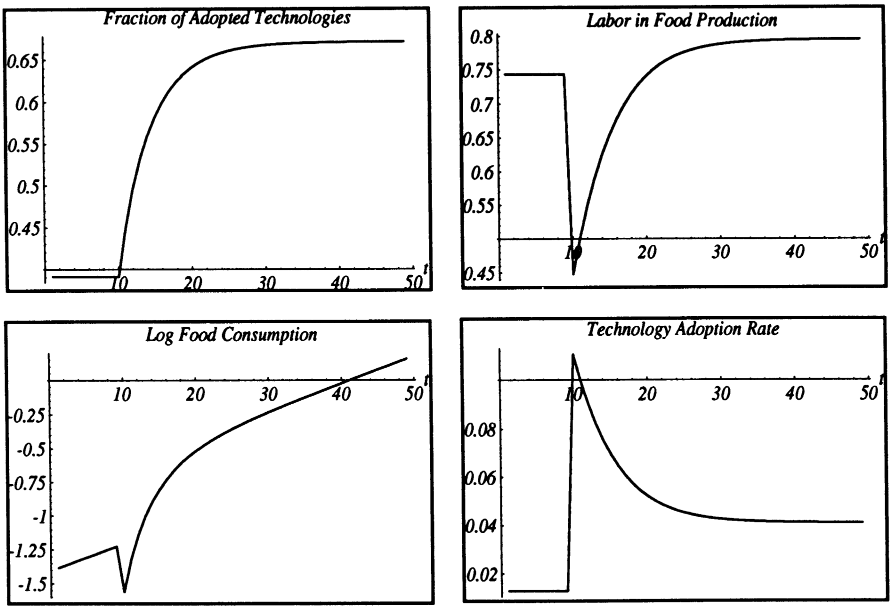
$$\frac{\dot{a}}{a} = \delta(1-u)\frac{(1-a)}{a} - \nu$$

which depends on the control u. Therefore we conduct the simulation only for a special case, namely $\sigma = 1$. The system's differential equations for this case are derived in appendix A.16 as:

$$\begin{aligned} \frac{\dot{a}}{a} &= \delta(1-u)\frac{(1-a)}{a} - \nu \\ \frac{\dot{u}}{u} &= \frac{\eta\delta u}{\alpha}\frac{(1-a)}{a} - \frac{\nu}{(1-a)} - \rho \\ \frac{\dot{n}}{n} &= \frac{(1-n)}{(1-\alpha n)}\left[-\rho - \mu + (1-\alpha)z_2(1-n)^{\alpha-1}\left(\frac{1-\gamma}{\gamma}\right)\right] \\ \frac{\dot{z}_2}{z_2} &= \mu - \alpha z_2(1-n)^{\alpha-1}\left(\frac{\gamma-n}{\gamma}\right) \end{aligned} \tag{5.23}$$

The equations from (5.23) show that the economy is dichotomous for $\sigma = 1$. A change in a variable only relevant for agriculture (a, u) does not influence the remaining variables (z_2, n) and vice versa. We can therefore analyze a change in δ by only considering the differential equations for a and u and applying the time-elimination method. We simulate an increase in δ from 0.05 to 0.2 which corresponds to a rise in a^* from 0.39 to 0.67, roughly the same as in last section. The results are plotted in figure 16.

Figure 16: Increasing Adoption Efficiency



First of all, one can observe that the transition period is now much shorter than with exogenous technology adoption. This reflects the fact that the economy can now temporarily increase the rate of adoption by devoting resources to this activity. The plots show that this happens indeed and that for a short time the adoption rate is increased. This labor shift into research is rather strong: the fraction of labor in research increases temporarily from about 25% to more than 50%. While this is probably unrealistically high, it only reflects the large, four-fold raise of δ. For the growth paths of c_A two effects exist. At first c_A decreases since less labor is engaged in actual food production. Later c_A increases again and eventually reaches a growth path higher than with a smaller adoption efficiency.

5.4 Summary

In this chapter we have modified the dual economy models from chapter 3 and 4 to analyze technology adoption instead of technology creation. Developing countries often have to adapt technologies from the industrialized North to suit their factor endowments and natural conditions. After being made compatible with production in the local country, these technologies have to be adopted by producers. This applies especially to agricultural techniques, not so much to industrial production.

The analysis has shown that in a model with technology adoption instead of technology creation the long-run growth rate of food consumption is exogenous to the South. Since the rate of technical progress in agriculture also influences the steady-state structure of the South, this structure is now influenced by the North. However, as we have seen in the numerical calculations from chapter 3, one should not expect these influences to be very large.

More important, consideration of technology adoption changes the effects of economic policy. While increases in ν and δ in the previous sections have increased the growth rates of food consumption, the equivalent policies, increases in the rate and efficiency of technology adoption m and δ, now have only level effects. Since level effects are always dominated by growth effects in the long-run, economic policy becomes less powerful in this context. The growth rate of food consumption is fully determined by the technological leader. In the endogenous adoption model an increase in δ also has a different effect upon u^*, the fraction of agricultural labor in food production, than in the endogenous technology creation model. While an increase in δ decreases this fraction in chapter 4, implying a larger fraction in research, it increases u^* in this chapter. Thus, the level of food consumption increases by two forces: the rate of technology adoption rises and a larger fraction of agricultural labor is employed in actual food production.

We therefore have to qualify the policy recommendations from chapter 4: an increase in the efficiency of research or schooling only leads to higher agricultural growth rates and more labor in this activity if the research process is at least partially creative and not only adaptive. Nevertheless, the level effects of adaptive research might well be worth the energy needed to obtain them.

The model also shows that full catch-up to technological leaders, that is, full convergence does never take place. Not in the exogenous adoption model since the rate of technology adoption is exogenously given, and not in the endogenous adoption model since technology adoption is costly. The model thus casts doubt on the central hypothesis behind the empirical convergence literature. While the problem of adoption is probably less severe in predominantly industrial economies, it is not negligible for most of the developing countries.

The model gives also a possible explanation for the empirical finding of "convergence clubs", convergence among countries in similar geographical regions or among similar countries like the OECD members (*Baumol*, 1986; *Durlauf* and *Johnson*, 1992). The need for technology adaptation arises especially in the agricultural sector and especially for countries who adapt technologies from countries with different soil or climate. If economies are predominantly industrial, like the OECD countries, this problem is less pronounced. If countries have similar climatic conditions, which is usually the case within a region, many agricultural technologies from other countries in this region can be put in use immediately.

6. Food Consumption and Economic Development

The previous chapters have focused on the *production* of agricultural goods and have analyzed those in exogenous as well as in endogenous growth frameworks. In this chapter we focus on the consumption side, especially on the influence food demand and consumption can have on the development process. The produce of the agricultural sector is crucially different from most industrial output. Food is necessary to stay alive, and under- or malnutrition can considerably impair a human's ability to work. While this may sound trivial and negligible from the point of view of an industrialized economy, such a relation can have important consequences for the development of an economy.

In this chapter we extend the exogenous growth model derived in chapter 3 to include two different aspects of food consumption. The first aspect is that there exists a subsistence requirement for food consumption. Consumption close to this level might have consequences for the individuals' behavior, especially for their intertemporal substitution decisions. The second aspect is that at low levels of nutrition a positive relationship between nutrition and the level of productivity exists. The two-sector model is especially suited for analyzing these topics. Its modification should shed some light on the question what consequences these effects can have for growth *and* industrialization in developing countries. Special attention is given to the question, whether the combination of these effects with a technologically stagnant agricultural sector can keep a country permanently from industrializing. While it would be desirable to introduce these effects also into the endogenous growth model, this would make the model intractable.[1]

6.1 Subsistence Consumption and Engel's Law

In this section the first consumption asymmetry is introduced into the two-sector model, namely Engel's law. *Engel* (1857) found in studies of household food consumption that with rising income the expenditure on food increased less than income: the income elasticity of food demand was less than unity. As a consequence, the share of food expenditure in total expenditure declined with rising income. This effect has also been observed on a macroeconomic scale for devel-

1. The reasons for this intractability are given below.

oped and developing countries alike and is one of the stylized facts mentioned in chapter three. *Houthakker* (1987, 143) points out that "of all empirical regularities observed in economic data, Engel's Law is probably the best established."

In all traditional dual economy models Engel's law is the driving force behind structural change. The intuition behind this mechanism goes as follows: an increase in agricultural productivity raises food output and thus the income of farm workers. However, due to the less than unitary income elasticity not all food is demanded by these workers. The surplus is therefore sold to industrial labor. The additional food supply decreases the relative price of food and thus increases the relative wage in the industrial sector. Migration of labor from agriculture to industry restores the equilibrium by decreasing food production and the industrial wage.

There exist different possibilities for modelling this effect. The first – which has been used in the classical dual economy models – is to explicitly state demand functions. *Jorgenson* (1961), for example, in the first neoclassical dual economy model simply assumed saturation at a certain level of income. Thus, the income elasticity for food demand beyond this point is zero. This strong assumption has of course been criticized for being unrealistic. It has been relaxed by *Zarembka* (1970) who assumed a Cobb-Douglas type consumption function with a constant income elasticity less than one.

The second possibility is to introduce a non-homotheticity into the utility function.[2] Since our models are based on utility maximization, this possibility is the appropriate one here. The simplest candidate for this purpose is the so-called Stone-Geary utility function:[3]

$$u(c_A, c_M) = \ln(c_A - \xi) + \ln(c_M)$$

In this function the income elasticity of food demand is given by (see appendix A.17) $\varepsilon_A = y / (y + \xi)$. This elasticity is less than one in the beginning and increases with rising income y, asymptotically converging towards unity. While this is not a very satisfactory behavior – it implies that Engel's law vanishes with

2. Recall that all homothetic utility functions imply an income elasticity of food demand which is unity (cf. *Varian*, 1984, 118ff.).
3. The function is named after *R. Stone* (1954) and *R.C. Geary* (1949) who have first used it.

rising income – the Stone-Geary function is the only simple and easily interpretable utility function generating Engel's law and does so at least for low income levels.[4] The elasticity of widget demand shows exactly the opposite behavior. The Stone-Geary function has the additional advantage that the non-homotheticity has an intuitive explanation: the term ξ can be interpreted as subsistence consumption requirement.

Rebelo (1992) as well as *King* and *Rebelo* (1993) have discussed the implications of a one good version of this function for economic development and the saving behavior over time. They have pointed out that the Stone-Geary function has an additional interesting property: Its intertemporal elasticity of substitution depends on the level of consumption. This elasticity, which is a measure for willingness and ability of consumers to shift consumption over time, is defined as $\sigma = -u'(c) / u''(c)c$ where c denotes the level of total consumption (cf. *Blanchard* and *Fisher*, 1989, 38ff.). For the above utility function these values are for food and widgets, respectively:

$$\sigma_A = 1 - \frac{\xi}{c_A}, \qquad \sigma_M = 1$$

Thus, the closer a country's food consumption is to subsistence consumption, the lower its intertemporal rate of substitution. This is intuitively clear: close to subsistence consumption, individuals are not able or willing to defer large amounts of consumption into the future, that is, to save a large part of their income. Note that the elasticity for widgets is unity. This differing ability to shift consumption of food and widgets over time is the fundamental asymmetry in this economy.

We can therefore observe all values of the intertemporal elasticity of substitution which we have used in simulations in the previous chapters over time if income rises. This allows us to specify a rather simple utility function. For the analysis conducted here we will therefore assume logarithmic utility and, in addition, substitute $c_A - \xi$ for c_A. Then the instantaneous utility function becomes:

$$u(c_A, c_M) = \gamma \ln(c_A - \xi) + (1-\gamma)\ln(c_M), \qquad \xi > 0 \tag{6.1}$$

4. *Atkeson* and *Ogaki* (1990) have also criticized this behavior as being contradicted by the data. They propose a modification of *Houthakker*'s (1960) utility function as an alternative. Since the non-homotheticity in the latter is difficult to interpret, however, the Stone-Geary function is used here.

This is very close to the Stone-Geary function, although it contains the additional parameter γ denoting the weight on food consumption. With this modification the income elasticity of food demand becomes $\varepsilon_A = y\gamma / (y\gamma + (1-\gamma)\xi)$. The intertemporal elasticities of substitution remain unchanged.

We will use this utility function to discuss the properties of the baseline dual economy model derived in chapter 3, when Engel's law is taken into account. The main topic are effects of economic policy in this setting. Of special interest are consequences of an increase in the agricultural rate of technical progress. Recall that it lead to a shift of labor from industry to agriculture in the steady-state, which, although quantitatively small, contradicts the empirically observed large shift in the other direction. Because Engel's law is such a well-established fact, its absence in the model might be responsible for the counter-factual outcome.

Assuming as before a constant labor force normalized to unity and, in addition, the Stone-Geary utility function (6.1), the problem (3.4) changes into:

$$(6.2)\quad \max_{n,\, c_M} \int_0^{\infty} (\gamma \ln (An^{\alpha} - \xi) + (1-\gamma) \ln (c_M))\, e^{-\rho t} dt$$

$$\text{s.t.} \quad \dot{K} = MK^{1-\alpha}(1-n)^{\alpha} - c_M$$

This leads to the following current-value Hamiltonian and its solution equations:

$$(6.3)\quad H_c = \gamma \ln (An^{\alpha} - \xi) + (1-\gamma) \ln (c_M) + \theta (MK^{1-\alpha}(1-n)^{\alpha} - c_M)$$

$$(6.4)\quad \frac{\partial H_c}{\partial c_M} = (1-\gamma) c_M^{-1} - \theta = 0$$

$$(6.5)\quad \frac{\partial H_c}{\partial n} = \alpha\gamma \frac{An^{\alpha-1}}{An^{\alpha} - \xi} - \alpha\theta MK^{1-\alpha}(1-n)^{\alpha-1} = 0$$

$$(6.6)\quad \frac{\partial H_c}{\partial \theta} = \dot{K} = MK^{1-\alpha}(1-n)^{\alpha} - c_M$$

$$(6.7)\quad \dot{\theta} = \theta\rho - \frac{\partial H_c}{\partial K} = \theta\rho - \theta(1-\alpha) MK^{-\alpha}(1-n)^{\alpha}$$

Equations (6.4) – (6.7) together with the familiar boundary conditions describe the economy's optimal path. The only equation that has changed substantially compared to the baseline model is equation (6.5) which now includes the subsistence term. This term is responsible for the absence of a "true" steady-state equilibrium. The first term in equation (6.5) will never be constant on the equilibrium path since $\xi > 0$. Therefore, a steady-state equilibrium cannot be derived. However, if A grows to infinity, the term will eventually tend to $1/n$. We will call this occurrence an "asymptotic steady-state".[5]

If the economy is at this asymptotic steady-state, its growth behavior as well as its structure is exactly the same as that of the baseline economy. It does, however, differ in levels.[6] The asymptotic steady-state is characterized by:

$$\left(\frac{\dot{c}_M}{c_M}\right)^* = \left(\frac{\dot{K}}{K}\right)^* = \frac{\mu}{\alpha}, \qquad \left(\frac{\dot{c}_A}{c_A}\right)^* = \nu$$

$$n^* = \frac{\gamma(\rho+\mu)}{\rho + (1-\gamma+\alpha\gamma)\dfrac{\mu}{\alpha}} \tag{6.8}$$

As we can see from (6.8), in the very long-run an increase in ν has no effect on the economy's steady-state structure (since $\sigma = 1$) but increases the growth rate of food consumption. Raising μ leads to a larger labor fraction in industry and increases the growth rate of widget consumption. The important difference to the baseline model is that now transitional dynamics become even more important since the asymptotic steady-state is only reached in the very distant future. If, for example, an economic policy increases the speed of transition towards a superior asymptotic steady-state, it might be worth considering; even if it does only marginally affect the asymptotic steady-state itself.

From the analytical solution alone, however, we cannot conclude anything about the time-scale of economic development. Equations (6.4) - (6.7) make up an interdependent system of differential equations which cannot be transformed into reduced form. Therefore we conduct numerical simulations of the transitional dynamics. For this purpose the modified time-elimination method will be employed. First, variables which are constant in the long-run (the asymptotic

5. This term may be misleading since a true steady-state is also asymptotic. However, the latter does not require that a variable grows to infinity and is therefore usually reached much earlier than the situation we call asymptotic steady-state.

6. These values are derived exactly as in the baseline model from chapter 3.

steady-state) have to be defined. As before, variables $z_1 = c_M / K$ and $z_2 = M / K^{\alpha}$ as well as n can be used. With this modifications the transformed equations (6.4) - (6.7) change into (cf. appendix A.18):

$$(6.9) \quad z_1 = \frac{(1-\gamma)}{\gamma} z_2 n (1-n)^{\alpha-1} \frac{An^{\alpha}-\xi}{An^{\alpha}}$$

$$(6.10) \quad \dot{z}_2 = z_2 \left(\mu - \alpha z_2 (1-n)^{\alpha-1} \left(\frac{An^{\alpha}(\gamma-n) + n\xi(1-\gamma)}{An^{\alpha}\gamma} \right) \right)$$

$$(6.11) \dot{n} = \frac{n(1-n)\left[(1-\alpha) z_2 (1-n)^{\alpha-1} \left(\frac{n(1-\gamma)(An^{\alpha}-\xi)}{An^{\alpha}\gamma} \right) - \rho - \mu - \frac{\nu\xi}{An^{\alpha}-\xi} \right]}{(1-\alpha n) + \alpha(1-n)\frac{\xi}{An^{\alpha}-\xi}}$$

Equation (6.9) gives the value for z_1 at every moment depending on the other two variables. Equations (6.10) and (6.11) describe how the economy evolves over time. However, all three equations depend on A, the state of productivity in agriculture, which grows exogenously over time. The system is therefore still time-dependent. We have already met such a problem in chapter 5 and we will solve it in the same way here. We start with $A_0 = 1$ and a value for z_{20} and calculate the policy function from equations (6.10) and (6.11) conditional on the value for A. This policy function describes the stable trajectory leading the economy to the point which would be the steady-state if A remained at A_0. It yields the optimal value for n, given A_0 and z_{20}. Then these steps are repeated for the next period where A has been increased exogenously and z_2 according to equation (6.10).[7] Subsequently the policy function is calculated again. Each increase in A brings the economy closer to the asymptotic steady-state and matches its dynamics better to those in the baseline economy since the term $(An^{\alpha}-\xi) / (An^{\alpha})$ vanishes. These calculations are done for each period up to the simulation horizon.

7. If technical progress in the agricultural sector were endogenous, this method could not be used since A is endogenous in this case. Thus, a clear distinction between movements on the trajectory towards the steady-state (the policy function) and shifts of the trajectory itself (the increase in A) are not possible any more. Any change in u on the trajectory would change the shift pattern of the trajectory itself. Thus the modified time elimination method cannot be used any more.

This method is used to conduct a few policy experiments where the rate of technical progress in either sector is increased. In all simulations does the economy begin with a stagnant agriculture, that is, $\nu = 0$. The state of technology, A, is normalized to one at this stage. Then the steady-state of this agriculturally stagnant economy can be derived by setting equations (6.10) and (6.11) equal to zero as:[8]

$$(6.12)\qquad z_1^{**} = \frac{\rho + \mu}{(1-\alpha)}, \qquad z_2^{**} = \frac{(\alpha\rho + \mu)}{\alpha(1-\alpha)\,(1-n^{**})^{\alpha}}$$

$$\frac{n^{**\,1-\alpha}\,(n^{**\,\alpha} - \xi)}{(1-n^{**})} = \frac{\alpha\gamma}{(1-\gamma)}\,\frac{(\rho+\mu)}{(\alpha\rho+\mu)}$$

Equations (6.12) yield the effects of subsistence consumption. Although the last equation cannot be solved for n^{**} analytically, the effects can be obtained by calculating the derivatives of the left-hand side term with respect to ξ and n^{**}. The former is negative and the latter positive.[9] Hence, an increase in ξ decreases the left-hand side term which must be increased again (since the right-hand side term is a constant) by an increase in n^{**}. In this model it is the subsistence consumption requirement together with a technologically stagnant agricultural sector that keeps the economy at a low degree of industrialization. Only with $\xi = 0$ does the economy reach the steady-state given by (6.8).

We now simulate the effects of some economic policies on this economy. For the simulations a value of $\xi = 0.3$ is chosen. With $\mu = 0.01$, $\rho = 0.05$, and $\alpha = 0.7$ this implies a fraction of labor in agriculture in the beginning of $n^{**} = 0.537$ which is not too far from the observed values mentioned in chapter 3.[10] The two possible policies are: (i) an increase in the rate of technical progress in agriculture (figure 17) and (ii) an increase in the rate of technical progress in industry (figure 18).

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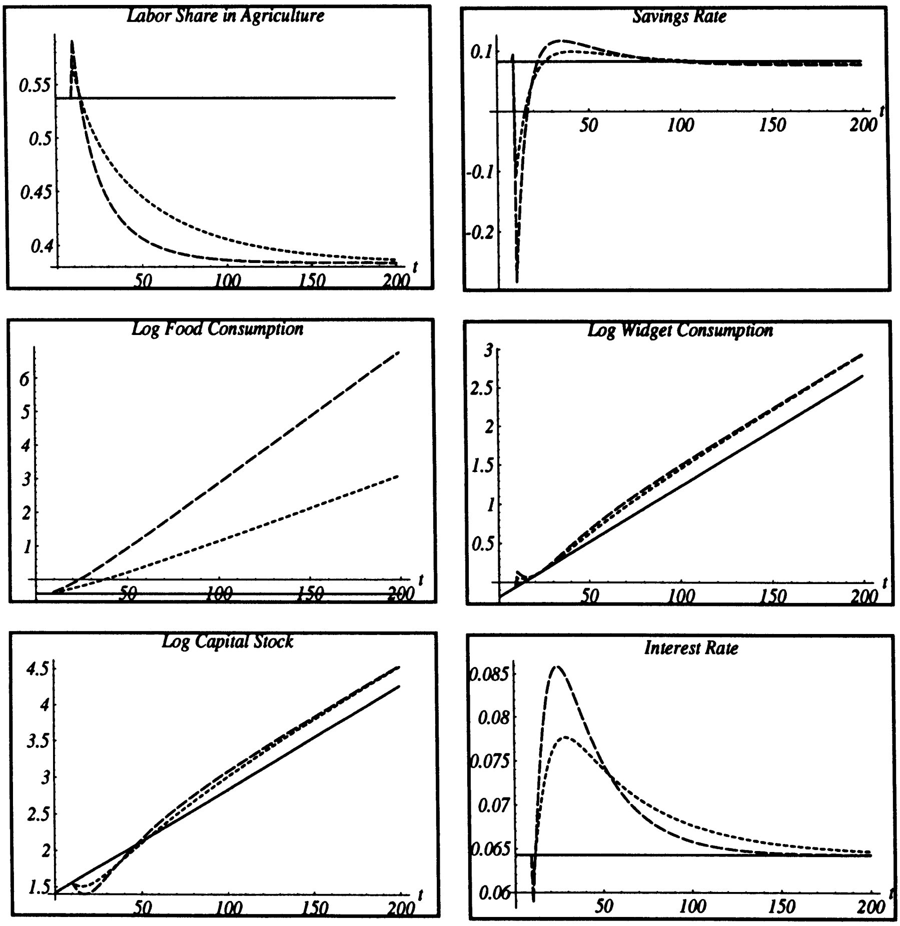
8. We characterize the steady-state with stagnant agriculture by a two-star superscript.
9. The first sign is trivial. The derivative of the left-hand side term with respect to n^{**} is given as:

$$\frac{\alpha}{1-n^{**}} + \frac{n^{**\,1-\alpha}\,(n^{**\,\alpha}-\xi)}{(1-n^{**})^2} + \frac{n^{**\,1-\alpha}\,(n^{**\,\alpha}-\xi)}{(1-n^{**})\,n^{**\,\alpha}}$$

which is positive if n^{*} is positive and the economy produces enough food for its subsistence.
10. Higher values of ξ would of course imply a larger fraction of labor in agriculture at the outset. However, a larger ξ leads to numerical problems in solving the model.

Figure 17: Increasing Agricultural Technical Progress



Legend: solid line: no change; dotted line: ν = 0.02; dashed line: ν = 0.04.

Figure 17 shows that the transitional dynamics do indeed last a long time in this model. With ν = 0.02 it takes the economy ca. 200 years to decrease its agricultural labor share from about 0.55 to 0.4. If economic policy increases this rate to ν = 0.04, industrialization happens much faster: the same shift happens in around 100 years – doubling the rate of technical progress in agriculture cuts the time needed for industrialization in half.

This simulation shows that the introduction of subsistence requirements into the exogenous growth model makes it far more realistic. Since the transitional dynamics last for several generations, they are certainly not negligible which casts some doubt on the steady-state focus of growth theory. Note that this duration is a rather conservative estimate: First of all, $\sigma = 1$ in the utility function is a rather low value, while chapter 3 has shown that a larger σ in most cases increases the transition duration. Also the subsistence requirement has been chosen rather low. A larger ξ would lengthen the transition dynamics by increasing the gap between first-period and steady-state fraction of labor in agriculture. While the (asymptotic) steady-state degree of industrialization is still determined by technologies and preferences (especially γ, as chapter 3 has shown) the speed of development is influenced by the rate of technical progress in agriculture.

The development of most of the other variables is mainly determined by the increase in agricultural productivity due to the raise in ν. This jump start raises the marginal utility from working in agriculture (the left-hand side of equation (6.5)) leading to a shift of labor into this sector. This decreases industrial output and, since savings can only come from industry, also the savings rate[11] and capital accumulation. This gap in the stock of capital compared to its steady-state growth path together with the shift of labor into industry led to the hump-shaped pattern in the interest rate, the marginal product of capital. Both increase the latter until the capital stock returns to its steady-state growth path.

While it is debatable if the exact pattern of development in figure 17 is realistic, its central message seems plausible: to behave optimally after an increase in the rate of agricultural technical progress, an economy first has to increase its labor force in agriculture to make use of the new opportunities. This short-run behavior is the opposite of the long-run effect. Subsequently the economy has to increase its savings rate to accumulate sufficient capital for the labor force which is now migrating to industry.

Figure 18 shows the consequences of an increase in the rate of industrial technical progress. This policy has only a very small effect on the degree of industrialization. The steady-state effect is only a slight decrease of agricultural labor as already discussed in chapter 3. Even with $\mu = 0.04$ this effect remains small compared to that observed in figure 17.

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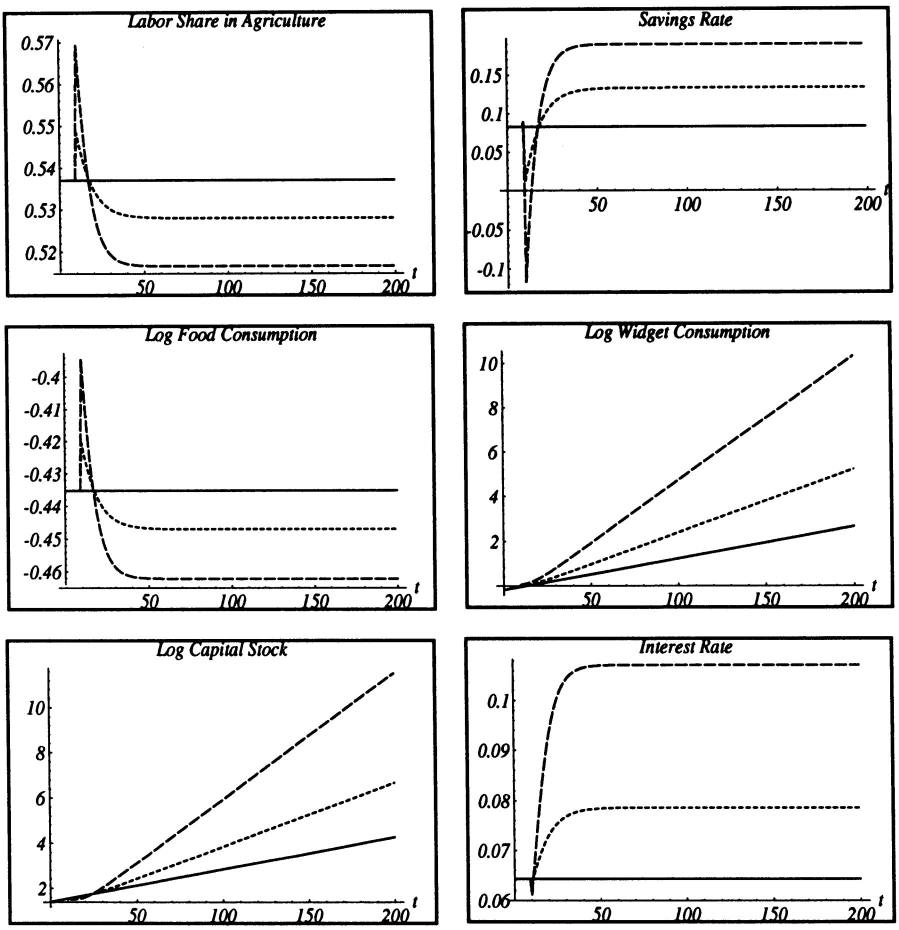
11. The savings rate is calculated here as investment over output of the industrial sector. Since we do not have relative prices, no GDP and therefore no savings rate for GDP can be calculated.

Figure 18: Increasing Industrial Technical Progress



Legend: solid line: no change; dotted line: $\mu = 0.02$; dashed line: $\mu = 0.04$.

This outcome is the result of the asymmetric consumption behavior for agricultural and industrial goods. This asymmetry also shows in the time paths for consumption. While an increase in ν raises the level of widget consumption and the growth rate of food consumption, an increase in μ only has positive effects in its own sector, where the growth rates of widget consumption and of capital accumulation rise. Also the savings rate is increased. The level of food consumption is even decreased by this policy, although this effect is only small. This outcome leads to the conclusion that an increase in the rate of technical progress in industry does not seem to be an adequate economic policy to foster structural change from an agricultural to an industrial economy. Instead it implies that a rise in the rate of agricultural technical progress is far more promising. Besides increasing the rate of growth of food consumption it leads via Engel's law to a more industrialized economy and thus also to an increase in the level of widget consumption.

Overall, the introduction of a subsistence consumption requirement has made the model far more realistic. It is now able to explain the gradual shift of labor towards industry, even when the necessary change in preferences has happened quickly. Also the time-scale of industrialization is by and large in the right dimension if one compares the simulation results with the second stylized fact. The model is thus able to replicate all three stylized facts mentioned in section 3.1.

It has also been shown that increases in the rate of agricultural technical progress can drastically accelerate the industrialization process. This policy might be one of the factors behind the success stories of countries like Taiwan, Korea, or Japan, all showing high GDP growth rates after productivity increasing agricultural reforms (*Grabowski*, 1993, 1994).

6.2 Productivity Effects of Nutrition

In this section a second asymmetry is introduced into the baseline model: a positive relationship between nutrition and productivity. This kind of relationship has recently found more interest, both theoretically (*Dasgupta*, 1993)[12] as well as empirically (*Fogel*, 1994). The idea of a technically determined relationship at low levels of income between the state of nutrition or health and labor productivity is familiar to development economists. It has been developed independently in the late 1950s by *Leibenstein* (1957) and *Mazumdar* (1959) and became soon known in development economics as "Efficiency Wage Hypothesis". It gained popularity in economics when *Stiglitz* (1976) made the first step to generalize the idea to the today under this label subsumed links between wages and efficiency in terms of incentives, morale and effort-intensity.[13]

For developing countries a number of empirical studies exist that estimate the influence of nutrition or other nutrition-related health indicators on labor productivity, mostly in agriculture. *Behrman* and *Deolalikar* (1988) review this litera-

12. *Dasgupta*'s main points are summarized in *Srinivasan* (1994b).

13. For a short review of this development see *Bardhan* (1993).

ture and note a general positive relationship between nutrition and productivity, although they criticize that several of these studies seem to suffer from methodological problems caused by self-selection bias or health endogeneity. But also studies without major methodological problems seem to support a positive relationship between nutrition and productivity.

Recently, *Fogel* (1994) made an attempt to estimate the importance of this nutrition effect for the development process of Britain. Rather than confining his work to the agricultural sector, he estimated the effects for the whole economy including industry. He concludes that improvements in gross nutrition account for 30% of the increase in per-capita income between 1790 and 1980. *Fogel* assigns one third of this effect to increased labor force participation, and asserts that this rise had been caused by improved nutrition which had strengthened the population and thus brought people into the labor force who previously were too weak to work. The remaining two thirds of the growth effect are said to be due to an increased labor productivity in production.

Fogel recalls that especially the poor have been too weak for intense work at the beginning of the industrial revolution: Around 1790

> [i]n France the bottom 10 percent of the labor force lacked the energy for regular work, and the next 10 percent had enough energy for less than three hours of light work daily (0.52 hours of heavy work). Although the English situation was somewhat better, the bottom 3 percent of its labor force lacked the energy for any work, but the balance of the bottom 20 percent had enough energy for about 6 hours of light work (1.09 hours of heavy work) each day.” (*ibid.*, 373)

But also the wealthier part of the society seems to have been far less healthy than today:

> [E]ven persons in the top half of the income distribution in Britain during the 18th century were stunted and wasted, suffered far more extensively from chronic diseases at young adult and middle ages than is true today and died 30 years sooner than today. (*ibid.*, 383)

Here these nutritional factors are introduced into the dual economy model derived in chapter 3. In addition to the influence of nutrition on labor productivity in production (henceforth called static nutrition effect) we also consider the influence on the ability to increase productivity (dynamic nutrition effect). The importance of dynamic effects has already been pointed out in chapter 4. Contrary to the model presented there, the dynamic effect here is modelled as occurring via a learning by doing mechanism (cf. section 2.1) which makes the model simpler and avoids the computational problems mentioned in this chapter's footnote 7. Both, the static as well as the dynamic nutrition effect are compared to situations where either malnutrition keeps productivity permanently below its maximum or where no nutrition-productivity relationship (NPR) exists. This comparison should yield some insights about the influence of different NPRs on the growth and development process. We also would like to know whether the existence of such a relationship together with a technologically stagnant agriculture can explain a large agricultural sector as it did in the last section.

A distinct feature of the NPR discussed here is that it exists only at low levels of nutrition. In fact, some empirical studies even show a negative relationship at higher levels of nutrition. *Strauss* (1986), for example, who analyzed farm households in Sierra Leone, estimated output elasticities of per-consumer equivalent calorie availability in agricultural production and found this elasticity to be 0.33 at the sample mean level of family calorie availability, 0.49 at 1500 calories per day, and 0.12 at 4500 calories per day. Above a daily consumption of 5200 calories, the estimated elasticity was negative. This fading of the NPR with rising nutrition makes a steady-state discussion of its effects impossible. When the economy finally grows with a constant rate, the nutrition effect has already petered out. Therefore we resort to numerical simulations to discuss the consequences of such an effect.

The remainder of this section is organized as follows: we first present some model modifications capturing different nutrition effects. In a second step a policy experiment similar to that from previous section is conducted and simulated numerically.

6.2.1 The Models

A relation between nutrition and productivity can exist in several different ways: First of all the NPR might be of a static nature, that is, an increase in food consumption increases labor productivity in production. This relationship might exist in agricultural as well as in industrial production. Here these possibilities are discussed separately to keep their effects as clear as possible. Combining such a relationship in agriculture with one in industry, while certainly more realistic, would only lead to superposition of their effects.

A further possibility is a dynamic relationship where an increase in food consumption raises the productivity growth rate. This could, for example, happen via the learning or schooling process since malnutrition not only reduces physical ability but also impairs mental capabilities. Therefore the model is extended to include the simplest possible element of endogenous productivity improvements, namely learning by doing. The outcome of this learning by doing process then depends positively on the level of food consumption. NPRs in learning by doing are of course only a very crude approximation of the true dynamic effects of malnutrition. Empirical evidence suggests that the main effects of malnutrition occur early in life. *Glewwe* and *Jacoby* (1995), for example, show that early childhood malnutrition causes delayed school enrollment. *Pollitt* (1984, 1990) reviews studies showing that children with severe malnutrition prior to school enrollment perform significantly worse on intelligence tests than better-nourished children. If, however, such effects last beyond school age, the grown-up children, who were malnourished as infants, will also perform worse in activities like technology adoption or learning.

To derive the model modifications, the NPR has to be specified first. From the empirical evidence it is clear that the effect should become less important as income increases. While this evidence describes relatively well the upper part of the functional form, its lower end is not clear. *Stiglitz* (1976) hypothesizes a logistic functional form but acknowledges that direct empirical evidence is difficult to obtain and therefore the functional form remains an open question. *Dasgupta* (1993) proposes a concave functional form. Since the latter is easier to analyze, we follow *Dasgupta* in defining nutrition caused productivity in the following way:

$$(6.13)\quad \Pi(c_A) = \frac{\pi c_A}{\pi + c_A}, \qquad \pi > 0$$

In this function productivity Π increases with rising food consumption c_A but is bounded from above by π. It shows diminishing returns to food consumption and does not possess a convex region like the logistic function.

Consider first a nutrition-productivity relationship in the agricultural sector itself. Assume that the agricultural production function takes on the form[14] $c_A = A\Pi n^{\alpha}$. As an additional restriction it is assumed that the NPR has the character of an externality. The agent does not take into account in her optimization that increased food consumption raises her productivity and thus the amount of food available for consumption. Rather, she assumes that Π grows exogenously.[15] The outcome is of course not any more the optimal solution to the agent's optimal control problem given by (as before the labor force is assumed constant and normalized to unity):[16]

$$(6.14)\quad \max_{n,\, c_M} \int_0^{\infty} \frac{\left[(A\Pi n^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} e^{-\rho t} dt$$

$$\text{s.t.}\quad \dot{K} = MK^{1-\alpha}(1-n)^{\alpha} - c_M$$

However, the result of this problem is still a dynamic equilibrium which can be interpreted as in chapter 3: The agent assumes an exogenous path for productivity Π If her expectations are met, and if the actual development of Π coincides with the expected development, then the economy is said to be in a dynamic equilibrium. Formalized this economy is characterized by the Hamiltonian

$$(6.15)\quad H_c = \frac{\left[(A\Pi n^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} + \theta\left(MK^{1-\alpha}(1-n)^{\alpha} - c_M\right)$$

---

14. A more reasonable assumption would be a purely labor augmenting nutrition effect. However, this would make the problem intractable.

15. The justification for this assumption is mainly simplicity. The optimal solution to this problem becomes too complicate to be tractable – even numerically – especially for the third case of a dynamic nutrition effect.
It is also intuitively clear what the differences between optimal and market solution should be. Since the latter neglects the productivity enhancing effect of nutrition and thus of food production, the market will allocate less labor than optimal to agriculture during the period where this effect is relevant.

16. Thus, the model only considers the productivity rising effect of better nutrition, not the labor force participation effect observed by *Fogel*.

with the following solution equations:

$$(6.16)\quad \frac{\partial H_c}{\partial c_M} = (1-\gamma)\left[(A\Pi n^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} c_M^{-1} - \theta = 0$$

$$(6.17)\quad \frac{\partial H_c}{\partial n} = \gamma\alpha\left[(A\Pi n^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} n^{-1} - \alpha\theta M K^{1-\alpha}(1-n)^{\alpha-1} = 0$$

$$(6.18)\quad \frac{\partial H_c}{\partial \theta} = \dot{K} = MK^{1-\alpha}(1-n)^{\alpha} - c_M$$

$$(6.19)\quad \dot{\theta} = \theta\rho - \frac{\partial H_c}{\partial K} = \theta\rho - \theta(1-\alpha)MK^{-\alpha}(1-n)^{\alpha}$$

where $\Pi = \pi(1 - 1/(An^{\alpha}))$.[17]

Equations (6.16) - (6.19) together with the familiar boundary conditions describe the dynamic equilibrium. However, contrary to the basic model from chapter 3 there does not exist a steady-state solution since Π neither remains constant nor grows without bound. Rather, if A grows forever, Π asymptotically converges towards its upper limit π. As in the previous section, there exists an asymptotic steady-state whose properties can be analyzed as before: If $\nu > 0$, Π will eventually be close to its upper limit π. Then equations (6.16) and (6.17) become:

$$\frac{\partial H_c}{\partial c_M} = (1-\gamma)\left[(A\pi n^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} c_M^{-1} - \theta = 0$$

$$\frac{\partial H_c}{\partial n} = \gamma\alpha\left[(A\pi n^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} n^{-1} - \alpha\theta M K^{1-\alpha}(1-n)^{\alpha-1} = 0$$

Comparing this outcome with the result of the baseline model, one can see that the asymptotic steady-state of a model including a nutrition-productivity relationship equals the steady-state of a model without.[18] This implies that in the very long run there is no difference in growth rates of the two economies, and even the structures (in terms of fractions of the labor force in agriculture) are identical. There might be a difference, however, in levels of consumption and capital as well as in the growth and development experience on the equilibrium path towards the

17. This is simply a transformation of: $\Pi = (\pi A \Pi n^{\alpha})/(\pi + A\Pi n^{\alpha})$.

18. The parameter π vanishes when deriving the steady-state in the same way as above.

(asymptotic) steady-state. Since equations (6.16) - (6.19) are a system of interdependent (differential) equations, though, an algebraic discussion of these differences is again not feasible.

Next, consider a nutrition productivity relationship existing only in industrial production. For this sector we can make the more reasonable assumption that Π works in a labor augmenting way since there are no analytical problems. After all the workers are the food consumers and become more productive. Then the industrial production function changes into

(6.20) $Y_M = MK^{1-\alpha}[(1-n)\Pi]^{\alpha}$.

If the agent does not take into account the NPR, the current-value Hamiltonian for her optimal control problem changes into:

(6.21) $$H_c = \frac{\left[(An^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} + \theta(MK^{1-\alpha}[(1-n)\Pi]^{\alpha} - c_M)$$

This current-value Hamiltonian leads to the solution equations:

(6.22) $$\frac{\partial H_c}{\partial c_M} = (1-\gamma)\left[(An^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} c_M^{-1} - \theta = 0$$

(6.23) $$\frac{\partial H_c}{\partial n} = \gamma\alpha\left[(An^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} n^{-1} - \alpha\theta MK^{1-\alpha}(1-n)^{\alpha-1}\Pi^{\alpha} = 0$$

(6.24) $$\frac{\partial H_c}{\partial \theta} = \dot{K} = MK^{1-\alpha}\Pi^{\alpha}(1-n)^{\alpha} - c_M$$

(6.25) $$\dot{\theta} = \theta\rho - \frac{\partial H_c}{\partial K} = \theta\rho - \theta(1-\alpha)MK^{-\alpha}\Pi^{\alpha}(1-n)^{\alpha}$$

where $\Pi = \pi A n^{\alpha}/(\pi + An^{\alpha})$.

This outcome is very similar to the previous model. The asymptotic steady-state can be derived as above. Again it is easy to see that in the asymptotic steady-state growth rates and structure of the economy are equal to those in the baseline model.

Finally, consider a dynamic nutrition-productivity relationship in the industrial sector.[19] In this case nutrition does influence the growth rate of productivity rather than its level. To keep the model as simple as possible, this relationship is assumed to work via a learning mechanism. The model follows *Arrow* (1962) in assuming that learning-effects are caused by capital accumulation.[20] The continuous introduction of new capital goods confronts labor continuously with new occasions to learn. Learning, in turn, increases the stock of knowledge which raises productivity. Following this idea, industrial production can be characterized as follows:

$$Y_M = MK^{1-\alpha}[(1-n)h]^{\alpha}$$

$$\text{(6.26)} \quad \text{where} \quad h = K^{\Pi} \quad \text{and} \quad \Pi = \frac{\pi A n^{\alpha}}{\pi + A n^{\alpha}}.$$

The level of human capital or knowledge is denoted by h and works in a labor augmenting way. Capital accumulation increases this knowledge with an elasticity Π This is where the NPR enters. It is assumed that workers learn better from the introduction of new capital goods when their nutrition level is higher. As before this effect is bounded from above.

Assuming again that the agent takes Π as exogenous, the current-value Hamiltonian from her optimization problem with h substituted by K^{Π} is:[21]

$$\text{(6.27)} \quad H_c = \frac{\left[(An^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} + \theta\left(MK^{1-\alpha(1-\Pi)}(1-n)^{\alpha} - c_M\right)$$

This Hamiltonian has the solution equations:

$$\text{(6.28)} \quad \frac{\partial H_c}{\partial c_M} = (1-\gamma)\left[(An^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} c_M^{-1} - \theta = 0$$

$$\text{(6.29)} \quad \frac{\partial H_c}{\partial n} = \gamma\alpha\left[(An^{\alpha})^{\gamma} c_M^{1-\gamma}\right]^{1-\sigma} n^{-1} - \alpha\theta MK^{1-\alpha(1-\Pi)}(1-n)^{\alpha-1} = 0$$

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19. The reason for not considering dynamic relationship in agriculture is mainly technical. Since there is no capital accumulation in this sector, the NPR would have to work via the stock of knowledge, very much as in chapter 5. Instead of engagement in technology adoption it would now be good nutrition which is necessary to put the new technologies in use. Simulation of this relationship, however, is not possible with the method used here since A is still time-depended but not any more exogenous.

20. Cf. section 2. 1.

21. *Mangasarian*'s sufficiency conditions are still met as long as $\Pi < 1$.

$$(6.30)\quad \frac{\partial H_c}{\partial \theta} = \dot{K} = MK^{1-\alpha(1-\Pi)}(1-n)^{\alpha} - c_M$$

$$(6.31)\quad \dot{\theta} = \theta\rho - \theta(1-\alpha)MK^{-\alpha(1-\Pi)}(1-n)^{\alpha}$$

with Π as in (6.26). In the same way as above the (asymptotic) steady-state values for growth rates and the fraction of labor in agriculture can be derived. These are slightly different now, which is due to the higher (total) rate of technical progress in industry:[22]

$$(6.32)\quad \left(\frac{\dot{c}_M}{c_M}\right)^{*} = \left(\frac{\dot{K}}{K}\right)^{*} = \frac{\mu}{\alpha(1-\pi)}$$

$$(6.33)\quad n^{*} = \frac{\gamma\left[\rho - (1-\sigma)\left(\gamma\nu + (1-\gamma)\frac{\mu}{\alpha(1-\pi)}\right) + \frac{\mu}{(1-\pi)}\right]}{\rho - (1-\sigma)\gamma\nu + \sigma(1-\gamma)\frac{\mu}{\alpha(1-\pi)} + \frac{\gamma\mu}{(1-\pi)}}$$

Having derived the models' solutions, the different growth and development paths described by the baseline model and the three variants from this section can now be compared by numerical simulation.

6.2.2 Transitional Dynamics

To simulate development in these models, the modified time-elimination method is used again. As an example, we derive the simulation equations for the second model, the static nutrition-productivity relationship in industry. The differential equations for the remaining two cases are obtained in the same way and can be found in appendix A.19. First, state-like and control-like variables have to be specified. For the second model these variables are $z_1 = c_M / K$ and $z_2 = M / K^{\alpha}$. As before, the third variable is n. Productivity Π at every point in time is given by

$$(6.34)\quad \Pi = \left(\frac{\pi A n^{\alpha}}{\pi + A n^{\alpha}}\right).$$

22. Therefore also the transversality condition changes into $\rho > (1-\sigma)\left(\gamma\nu + (1-\gamma)\frac{\mu}{\alpha(1-\pi)}\right)$.

Combination of equations (6.22) and (6.23) yields the familiar but slightly modified equation for z_1.

$$(6.35)\quad z_1 = \frac{(1-\gamma)}{\gamma} z_2 n \Pi^{\alpha} (1-n)^{\alpha-1}$$

Using the definition for z_2 and equation (6.24) yields the development of z_2 as

$$(6.36)\quad \dot{z}_2 = z_2 \left(\mu - \alpha z_2 \Pi^{\alpha} (1-n)^{\alpha-1} \frac{(\gamma-n)}{\gamma}\right).$$

Finally, the differential equation for n can be obtained from differentiating equation (6.22) and (6.34) while using equations (6.25), (6.35), and (6.36):

$$(6.37)\quad \dot{n} = \frac{(1-n)\left[(1-\sigma)(\mu(1-\gamma)+\gamma\nu)-\mu-\rho+\alpha\nu((1-\sigma)(1-\gamma)-1)(1-\frac{\Pi}{\pi})\right]}{n^{-1}(\sigma(1-\alpha n) + (1-\sigma)\gamma(1-\alpha) - \alpha^2((1-\sigma)(1-\gamma)-1)(1-\frac{\Pi}{\pi}))} + \frac{(1-\alpha)(1-\gamma) z_2 \Pi^{\alpha} \frac{(\sigma n + \gamma(1-\sigma))}{\gamma}}{\sigma(1-\alpha n) + (1-\sigma)\gamma(1-\alpha) - \alpha^2((1-\sigma)(1-\gamma)-1)(1-\frac{\Pi}{\pi})}$$

For each of these three cases, combination of the differential equations for n and z_2 yields the derivative of a single policy function. However, if the definition for Π is substituted into the equations to eliminate Π, the differential equation still depends on A which makes the problem time-dependent. To solve this problem, we apply the time-elimination method within a two-step approach. Starting with values for A_0 and z_{20}, the trajectory towards that (fictive) equilibrium where A would remain at A_0 is calculated in the first step. The outcome is a policy function $n(z_2, A_0)$, giving the optimal value for n. In the second step A is increased exogenously and z_2 according to its differential equation. Then the first step is conducted again. Eventually agricultural productivity A will be so large that the NPR disappears and the economy takes on the steady-state values from the baseline model.[23] The intuition behind this procedure is the following: in every period the stable trajectory conditional on A is calculated. For each A exists a different trajectory. The combination of movements on this trajectory (for n and z_2) and movements between trajectories (due to changes in A) describes the evolvement of the economy's variables over time.

To conduct the simulations, parameter values have to be chosen first. As before, we use $\rho = 0.05$, $\alpha = 0.7$, and $\sigma = 5$. The upper limit of the nutrition-productivity relationship, π, is set to unity for the static NPR and to $\pi = 0.05$ for the dynamic relationship. For the former, $\pi = 1$ corresponds to a situation without NPR. Since in the latter $\pi = 1$ is not feasible (in the steady-statethe growth rate $\dot{c}_M / c_M = \mu / \alpha (1 - \pi)$ would go to infinity) we choose π as well as μ half as large as in the static case implying the same steady-state growth rates of widget consumption as well as the same steady-state fractions of labor in agriculture.

With these parameter values we conduct a simulation similar to that from previous section. In the beginning, the agricultural sector is technologically stagnant. At some point in time the rate of technical progress in this sector becomes positive. We study the economy's behavior after this shock for two cases: normal agricultural technical progress ($\nu = 0.02$) and fast agricultural technical progress ($\nu = 0.04$). These two cases are simulated for each of the three possible nutrition-productivity relationships. In all cases does the rate of technical progress in industry remain constant at $\mu = 0.02$. Industrial technical progress does not influence nutrition caused productivity but only leads to a new steady-state division of labor and a higher growth rate of widget consumption.

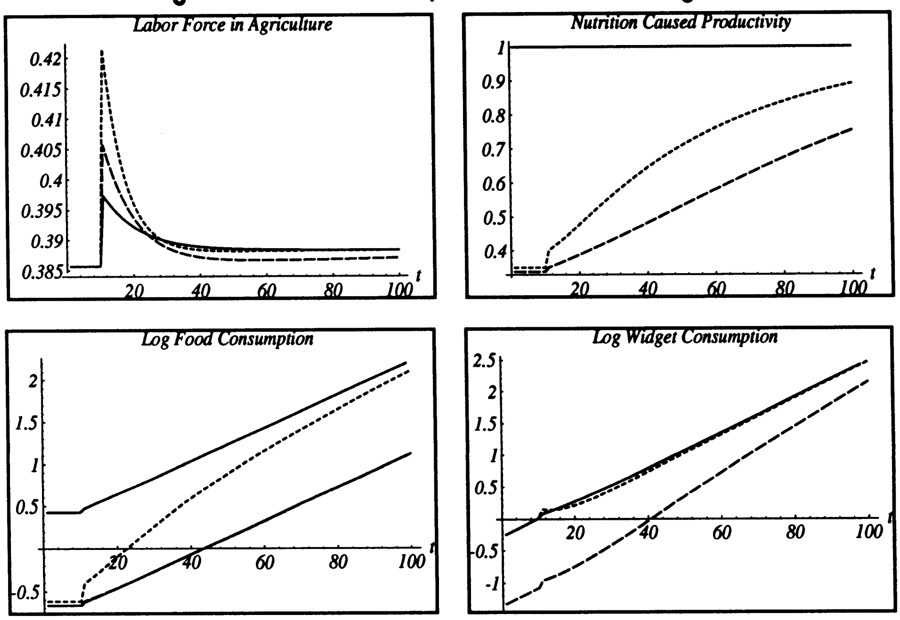
The outcome of these simulations is depicted in figures 19 - 22. Figures 19 - 20 contain the two static relationships in agriculture and industry while figures 21 - 22 show the dynamics of the two possible NPRs in industry.[24] The first ten periods show the pre-shock case where $\nu = 0$ and nutrition caused productivity remains permanently below its maximum value. The solid line in each picture describes the benchmark case, namely the path an economy would take when the nutrition caused productivity is always at its maximum π. This baseline economy from chapter 3 is exposed to the same shock in ν. A comparison of both paths

23. Strictly speaking, this occurs only in infinite time. But for a numerical solution it is sufficient to require that the difference be less than the precision used in solving the problem. For example, with a rate of technical progress in agriculture of 3% per year and a starting value of $A_0 = 1$ productivity Π converges to π within approximately 200 years. While this is longer than one would sensibly expect, equation (6.13) could be easily modified to converge towards its limit more quickly, for example by using $c_A{}^2$ instead of c_A. However, since this would only complicate the analytical parts of the solution while not yielding much new insight, equation (6.13) is left as above.

24. Note that the number of simulation periods has been chosen differently between the pairs to make the dynamics clearer.

shows the influence of the NPR on the pre-shock steady-state as well as on the transitional dynamics towards the new steady-state equilibrium. A further benchmark case also contained in the figures is the simple no-change scenario. Extending the time-paths from the first ten periods into the future yields the development of an economy where agriculture remains technologically stagnant.

Figure 19: Static NPRs, Slow Technical Progress

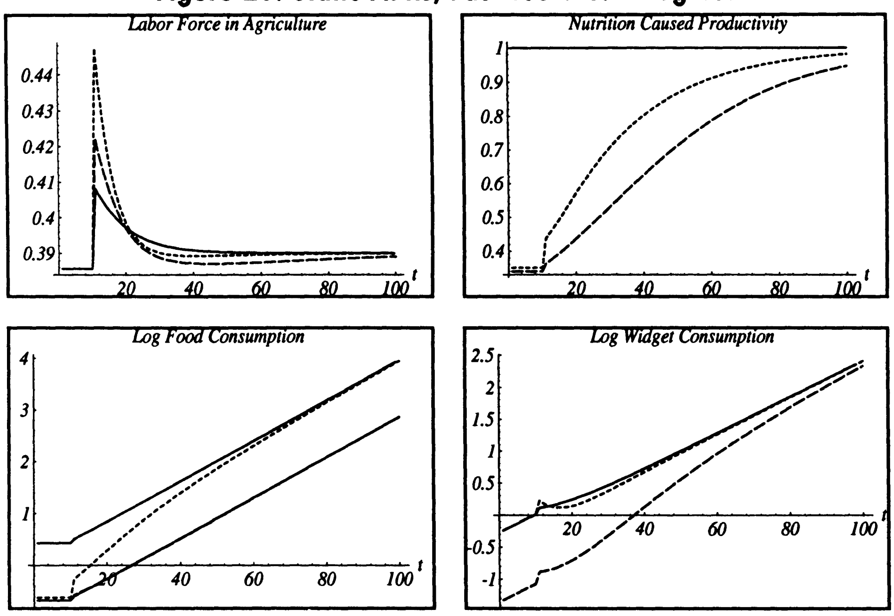


Legend: solid line: baseline model(s); dotted line: agricultural NPR; dashed line: industrial NPR.
Note: dashed and lower solid lines for food consumption coincide.

Consider the static relationships first. Figure 19 shows the slow progress case and figure 20 the time paths with fast agricultural technical progress. Although agricultural and industrial relationship are depicted together, they can only be compared very carefully. First of all the NPR is labor augmenting in industry but labor and land augmenting in agriculture. And secondly, different starting values for A had to be chosen due to computational problems: $A_0 = 1$ for the industrial and $A_0 = 3$ for the agricultural relationship.

Nevertheless, several observations can be made: First of all, the simulations show that agricultural stagnation together with the existence of nutrition-productivity relationships does not lead to a permanently low degree of industrialization. This contrasts the findings from previous section where we have shown that agricultural stagnation together with Engel's law does have this effect. Here no observable difference exists between the values for the baseline economy and those with a NPR. We can only observe an effect already derived in chapter 3, namely a rise of n after ν has been increased. An explanation for this difference between the two model extensions might be that the first directly affects utility while the second only affects production functions. We have already seen in chapters 3 and 4 that changes in preferences tend to lead to rather strong effects. We can also see from comparing figure 17 to figure 19, for example, that the introduction of subsistence consumption is fully borne by adjustments in n in the previous section, while the NPR leads to adjustments in the consumption paths, too.

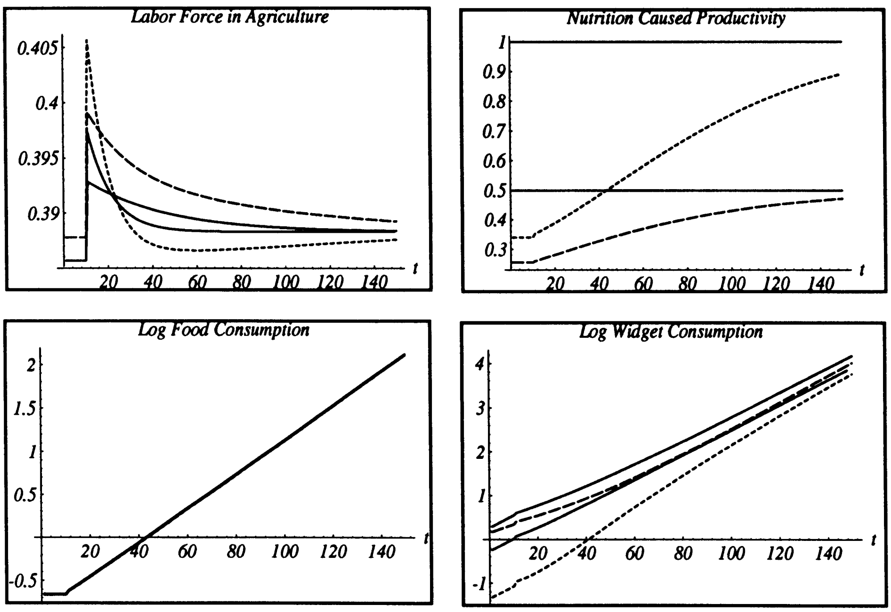
Figure 20: Static NPRs, Fast Technical Progress



Legend: solid line: baseline model(s); dotted line: agricultural NPR; dashed line: industrial NPR.
Note: dashed and lower solid lines for food consumption coincide.

Secondly, the simulations show that the fraction of labor in agriculture sharply increases right after the shock and subsequently decreases slowly toward its new steady-state. This is due to the suddenly increased marginal utility of using labor in agricultural production. For both cases it takes about 30 years until the economy is close to its new steady-state value for n. This is rather short compared to the time it takes the economy to reach its upper limit of nutrition caused productivity. The duration of the latter, however, is unrealistically long which is due to the functional form chosen.

Figure 21: NPRs in Industry, Slow Technical Progress



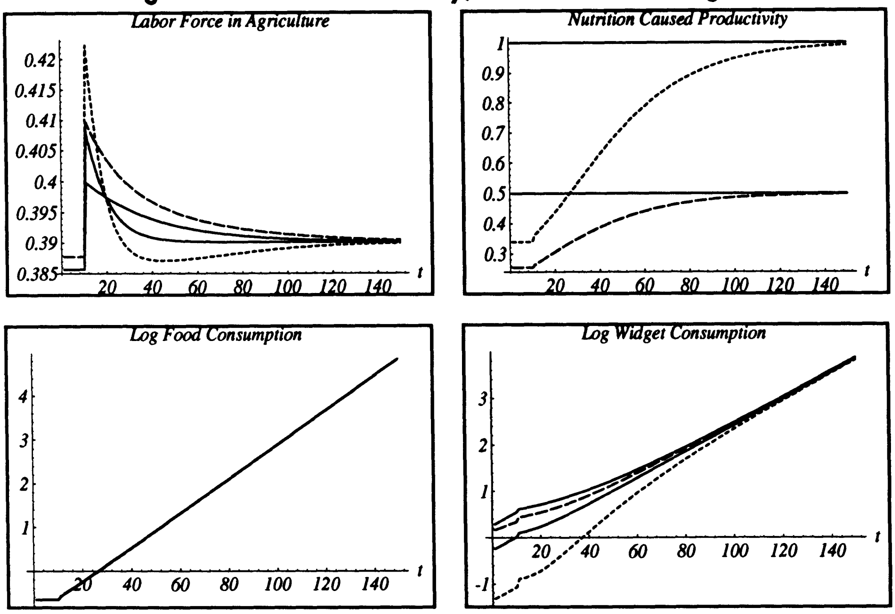
Legend: solid line: baseline model(s); dotted line: static NPR; dashed line: dynamic NPR.
Note: All lines for food consumption coincide.

The gap between dashed or dotted lines and the solid line(s) is the forgone consumption which is lost due to the fact that productivity was not always at its highest possible level. Although the structural effects are small, the output effects of both NPRs are rather large. The gap can be used to calculate the contribution of the NPR to the increase of widget consumption. For the static NPRs this contribution can be calculated as fraction of the initial gap between solid and dashed or dotted line to the total increase of consumption over the time period considered.[25] For industry this rough calculation yields a contribution of about 6% ($\nu = 0.02$) and 5% ($\nu = 0.04$) and for agriculture of 13% ($\nu = 0.02$) and 2% ($\nu = 0.04$). Most of the values are far below those obtained by *Fogel*, even more so

25. This assumes that nutrition caused productivity is at its maximum in the final period as is the case in figure 20.

since he has considered a time-period twice as long as ours and the contribution of the NPR decreases as the level of (overall) productivity rises.

Figure 22: NPRs in Industry, Fast Technical Progress



Legend: solid line: baseline model(s); dotted line: static NPR; dashed line: dynamic NPR
Note: All lines for food consumption coincide.

Figures 21 and 22 show the outcome for the static and dynamic NPRs in industry together. According to these plots the static NPR seems to have larger consequences since upper solid and dashed line are closer together than lower solid and dotted line in the plots for widget consumption. This reflects the influence of the static NPR on the *level* of widget production which does not exist in this extend for the dynamic relationship. Since the two levels of production deviate less in the latter scenario, the time-paths are closer together. However, in the long-run the dynamic relationship is more important. With static nutrition caused productivity permanently below its maximum, the growth path of widget consumption would run below but parallel to the growth path characterizing an economy where Π is at its maximum. With a dynamic NPR in such a scenario, though, the path would be lower and flatter. In figures 3 and 4 it would be given by extending the slope of the dashed path from the pre-shock period into the future. The gap between the two paths would thus be widening over time. In addition, the rate of technical progress as well as the highest possible value for nutrition caused productivity are only half as large in the dynamic case than in the static case. Using the same maximum value in the latter scenario would shift the growth path of widget consumption down to half of its present value. In addition cutting the rate of technical progress in half would also make the path flatter. Taking this into account, the dynamic nutrition-productivity relationship becomes even more important.

The contribution of the nutrition caused productivity increases to the raise in widget consumption can be calculated in the following way: extend the first ten years of the dashed line into the future. This line then describes the growth path of an economy with Π fixed below its maximum. Compare the level of food consumption under this scenario with that from the simulation with dynamic nutrition effect. After 100 years the contribution of better nutrition to the increase in widget consumption would be 55% ($\nu = 0.02$) or 50% ($\nu = 0.04$), respectively. 50 years later these numbers would increase to 165% ($\nu = 0.02$) and 130% ($\nu = 0.04$), respectively. These values are considerably larger than those obtained for the static effect and also larger than those obtained by *Fogel.*

6.3 Summary

In this chapter we have presented simple extensions to the dual economy model from chapter 3 that introduce asymmetries between food and widget consumption into the model. Despite their simplicity both lead to dynamics which cannot be analyzed anymore with purely algebraic tools. A numerical technique, the time-elimination method, has been extended to simulate explicitly the development of economies characterized by the mentioned asymmetries. Both, Engel's law as well as productivity effects caused by nutrition have a large influence on the transition path of an economy towards its steady-state. While the former lengthens this path compared to the baseline economy without this chapter's model extensions, the latter only changes its structure.

Both extensions have two things in common: first of all, in both models does an increase in the rate of agricultural technical progress have more desirable effects than a rise in industrial technical progress. This qualifies the previous results from chapter 3 which would rather support policies to increase industrial technical progress. Since at least the existence of Engel's law is undebatable, these chapter's models support the policy prescription made by the classical dual economy models, that increases in agricultural technical progress are a precondition for industrialization.

Secondly, both models show the usefulness of numerical simulations for the analysis of transitional dynamics. In both model modifications the long-run behavior of the models is identical. Despite these similarities both lead to totally different transitional dynamics and imply different interpretations.

There exists also one major difference between both extensions: Engel's law together with the absence of technical progress in agriculture can explain why an economy may be stuck in an underdevelopment trap of a permanently low degree of industrialization. An increase in agricultural technical progress can carry the economy out of this trap. In comparison, the existence of a nutrition-productivity relationship together with a technologically stagnant agriculture alone cannot explain such a permanently large agriculture. It does, however, lead to long-lasting effects of plausible size. The reason for this difference is that the modifications work via preferences in the first case and via the production function in the second. Similar to chapters 3 and 4 changes in preferences have stronger effects than changes in production.

The consequences of such a relationship depend very much on the sector in which it exists. A relationship in agriculture primarily influences the level of food consumption while one in industry influences mainly output of this sector. If the nutrition productivity relationship is static, these effects are only important for a certain length of time and become negligible as total factor productivity becomes large compared to nutrition caused productivity. In addition, malnutrition has only level effects.

This is different for a dynamic nutrition-productivity relationship where better nutrition increases the productivity of the learning by doing process. Such a relationship has not only level effects but also growth effects. In a malnourished economy the growth rates of consumption remain permanently below those possible with better nutrition. The contribution of better nutrition to consumption growth is several times larger for a dynamic NPR relationship than for a static one.

The contribution of nutrition-productivity relationships to the total increase in consumption of food or manufacturing goods implied by our model is considerably lower for the static relationships than stated by *Fogel* (1994). Under presence of a dynamic NPR, the model implies much larger contributions. These calculations have to be take with care, however. While *Fogel's* calculations are based on real data, the results of the model can at best form the basis for a calibration exercise. The results are subject to the length of the time period considered, the parameter values as well as the specific functional forms assumed.

7. Conclusions

In this study we have combined the neoclassical model of a dual economy with methods and research topics from the New Growth Theory to study the consequences of agricultural productivity growth for a two-sector economy. This analysis has produced some interesting results for economic policies in developing countries as well as for growth and development theory. A considerable part of these results has been obtained by employing numerical techniques which so far have only rarely been used within the framework chosen here. In addition to providing information about the quantitative effects of economic policies, these techniques have freed us from the straight-jacket of steady-state analysis, at least to a certain extend.

The central model has been developed in chapters 3 and 4. Chapter 3 contains an exogenous growth version and chapter 4 an endogenous growth model, although endogenous technical progress in the latter is limited to the agricultural sector. In both models development is understood as a transition between two steady-states. Part of this development process, namely structural change, comes to an end eventually. This set-up allowed us to divide the analysis of development into two parts, namely determinants of the final outcome and determinants of the path towards this outcome. Since the first part concerns the economy's steady-state and the second its transitional dynamics, the former could be analyzed mainly analytically while the latter has been examined numerically.

In both models does faster technical progress or more efficient research or human capital accumulation increase the growth rates of consumption of the respective good. While these are the positive effects one would expect, there do exist negative cross-effects as the endogenous growth model in chapter 4 has shown: An increase in industrial technical progress decreases the rate of technical progress in agriculture. Behind this result is labor migration between different activities: the higher productivity of industry makes it more attractive to work in this sector than in agriculture. Due to the labor shift towards industry does the remaining agricultural labor spend more time in production than before which decreases the rate of technical progress in agriculture. Although not modelled here, these mechanisms should work equally for increases in agricultural growth rates. A policy maker with the objective to increase growth rates should therefore take these side effects of his policy into account. Such negative cross-effects cannot arise in models of exogenous growth. They are therefore not discussed in the classical dual economy literature.

Both models show the same outcome for determinants of the economy's steady-state structure. In both cases a higher rate of technical progress in industry implies a more industrialized country while faster technical progress in agriculture leads to a less industrialized economy. The latter effect, however, depends very much on the utility specification used. If Engel's law, that is, a less than unitary income elasticity of food demand is introduced into the model as done in chapter 6, then increases in the rate of agricultural technical progress also lead to industrialization.

In addition it turned out that the quantitative effects of the rates of technical progress on the economy's structure are rather small; far too small to explain the empirically observed labor shifts between sectors by changes in rates of technical progress. The parameter which crucially determines the economy's structure is the weight of food in utility, thus preferences. Structural change in our model has to be preceded by a change in preferences, that is, by a change in demand parameters. While this is a necessary condition, it is not sufficient. We have shown in chapter 6 that a combination of Engel's law and a technologically stagnant agriculture can keep an economy in an underdevelopment trap with a lower degree of industrialization than in a comparative economy with a positive rate of agricultural technical progress.

In chapter 4 we have also introduced externalities of research or human capital accumulation in agriculture and have studied the consequences. As expected, these externalities lead to lower growth rates in the agricultural sector. This is equivalent to the outcome of standard endogenous growth models. According to the model this externality also leads to over-industrialization which, at first sight, seems to be counter-factual. Poor countries, which do have low growth rates in agriculture in most cases – at least partially due to sub-optimal investment in research or human capital – are not particularly over-industrialized. We have shown, however, that this counter-factual outcome is due to the neoclassical assumption of full-employment and that the outcome should better be interpreted as over-urbanization which we do indeed observe in Third World countries. Farmers underestimate the gains from staying in agriculture combined with, for example, investing in schooling and therefore migrate to the cities. Hence, policies aiming at realization of the socially optimal investment in research or human capital accumulation in agriculture not only increase the food growth rate but also counteract over-urbanization.

In both models do changes in the rates of technical progress also influence the transitional dynamics. In all cases do increases in these rates shorten the transitional period considerably. There is no unequivocal result, however, which effect is stronger. Since the influence of technical progress on the economy's steady-state is very small in comparison, the main advantages of higher rates of technical progress are thus a faster transition which in this framework is equivalent to a faster industrialization.

The two models have also led to some interesting technical results: in both cases the steady-state equilibrium is stable and unique, at least for a large range of realistic parameter values. This outcome differs from other research on properties of two-sector endogenous growth models which has found a quite frequent occurrence of multiple equilibria.

For both models an attempt has been made to fit them to stylized facts of economic development, especially stylized facts about the size of structural change and the duration of this process. The exogenous growth model – especially the version including Engel's law – fitted these facts quite well while the endogenous growth model showed too short transition periods due to the additional control variable. In addition, the steady-states in this model changed considerably for rather small modifications of parameter values. The result of endogenizing technical progress in agriculture is therefore contradictory: on the one hand the endogenous growth model can explain more phenomena and takes into account more interdependencies within the model. On the other hand, the quantitative results become less realistic, especially the transitional period shortens considerably. Since, however, in reality these variables *are* endogenous to a large extend, further research should focus on the counterpart of unrealistically quick adjustments, on mechanisms leading to sluggish adjustment.

Chapters 5 and 6 contain modifications of the models from chapters 3 and 4. In chapter 5 we have investigated whether the replacement of technology creation by technology adoption in agriculture changes the economy's behavior. It turned out that it did. The model builds on the observation made in the development literature that agricultural techniques from other countries can only rarely be used immediately. Usually, they have to be adapted to be adoptable. If technology is adopted from some technologically superior country, this country ultimately determines the growth rate of food consumption in the developing economy. Therefore economic policies aiming at improving the process of technology adoption have only level effects.

The model also shows that full catch-up to the technology levels of a leader almost never takes place. This result holds for the exogenous and the endogenous growth versions alike. In both cases would the rate of technology adoption have to go to infinity to achieve full technological convergence. In the endogenous adoption model not even the highest rate of technology adoption is chosen since this activity has the opportunity cost of less food production.

This outcome casts some doubt on the central assumption of the empirical convergence literature which infers from the existence of such a gap the necessity of a subsequent catch-up process. It also gives a possible explanation for the finding of convergence clubs, either with members from the same geographical region or with members on similar stages of industrialization. If an economy is predominantly industrial, or if economies have similar climatic conditions, the necessity to adapt agricultural technologies becomes less pronounced and the argument against convergence becomes weaker.

The last chapter has studied the consequences resulting from peculiarities of food consumption. We have extended the baseline model from chapter 3 by introducing either a less than unitary income elasticity of food demand or a relationship between the level of food consumption and productivity. In the latter model we analyzed 3 different possibilities for this relationship: it might occur in agricultural production, in industrial production, or in industrial knowledge accumulation. While Engel's law in conjunction with technological stagnation in agriculture implies a permanently low degree of industrialization of the economy, the negative productivity effects of mal- or undernutrition are not strong enough to be responsible for such an outcome.

In both models increases in the rate of agricultural technical progress are superior to those in industry if the policy maker aims at industrialization. In the first model with Engel's law it is agricultural technical progress which carries the economy out of the state of low industrialization. An increase in industrial technical progress only has a small effect on the economy's steady-state structure. The same is true for the second case where agricultural technical progress brings the economy towards the highest possible value of nutrition caused productivity. Again a change in industrial technical progress only influences the steady-state split of labor between the sectors and the growth rate of this sector. The size of the effects from a nutrition-productivity relationship depend very much on the question whether it is static or dynamic. While the numerical calculations yielded a contribution of the nutrition effect to the increase in per capita consumption over 100 years between 2% and 13% for a static relationship, this fraction was between 50% and 55% when nutrition was assumed to influence the growth rate of productivity.

Overall, the study conducted here has shown that the analysis of a two-sector economy with models and tools from the NGT is fertile and generates interesting results. Further research should therefore extend this analysis, for example by taking into account risk, financial markets or openness to trade and foreign investment. Also the use of numerical methods in conjunction with an analytical discussion has proved useful. Many of the results could not have been obtained from a purely analytical discussion. Since such a combination of techniques avoids problems from using numerical techniques alone and is able to explain more than a purely analytical treatment, it seems to be a promising road for further research.

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Mathematical Appendix

A.1 Baseline Model: Sufficiency Conditions

Mangasarian's sufficiency conditions are the following: For a problem

$$\max \int_0^{\infty} F(x, u)\, e^{-\rho t} dt$$

$$\text{s.t.} \quad \dot{x} = f(x, u)$$

where x denotes the vector of state variables and u the vector of control variables, the necessary conditions are also sufficient if $F(x, u)$ and $f(x, u)$ are both jointly concave in x and u and $\lambda \geq 0 \; \forall \; t$. A function $f(\mathbf{x})$ is concave on an open convex subset S in $\mathbf{R}^n$ if and only if for all $\mathbf{x} \in S$ and for all Δ_r, $(-1)^r \Delta_r(\mathbf{x}) \geq 0$ for $r = 1, \ldots, n$, where the principal minors $\Delta_r(\mathbf{x})$ of order r in the Hessian matrix $f''(\mathbf{x})$ are the determinants of the sub-matrices obtained by deleting $n - r$ arbitrary rows and then deleting the $n - r$ columns having the same numbers (*Berck* and *Sydsæter*, 1991)

For the problem (3.4) we have

$$F(K, n, c_M) = L \frac{\left[\left(\frac{A}{L} (nL)^{\alpha} \right)^{\gamma} c_M^{1-\gamma} \right]^{1-\sigma}}{1-\sigma},$$

$$f(K, n, c_M) = MK^{1-\alpha} ((1-n)L)^{\alpha} - Lc_M,$$

and $\lambda \geq 0$ by assumption.

For a function to be concave the Hessian determinant must be negative semidefinite, which is the case if the principal minors change signs. Consider first $f(K, n, c_M)$:

$$H_f = \begin{bmatrix} -\alpha(1-\alpha) MK^{-1-\alpha} ((1-n)L)^{\alpha} & \alpha(1-\alpha) MK^{-\alpha} L ((1-n)L)^{-1+\alpha} & 0 \\ \alpha(1-\alpha) MK^{-\alpha} L ((1-n)L)^{-1+\alpha} & -(1-\alpha)\alpha MK^{1-\alpha} L^2 ((1-n)L)^{-2+\alpha} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

It can be seen from H_f that two principal minors of order one are negative since $1 - \alpha > 0$ and the third one is zero. All other principal minors are zero, too. Thus $f(K, n, c_M)$ is concave in K, n, and c_M.

Next consider $F(K, n, c_M)$. For this equation the Hessian is:

$$H_F = L \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\alpha\gamma(\alpha\gamma(1-\sigma) - 1)\Delta}{n^2} & \frac{\alpha\gamma(1-\gamma)(1-\sigma)\Delta}{nc_M} \\ 0 & \frac{\alpha\gamma(1-\gamma)(1-\sigma)\Delta}{nc_M} & \frac{-(1-(1-\gamma)(1-\sigma))(1-\gamma)\Delta}{c_M^2} \end{bmatrix}$$

$$\text{where} \quad \Delta = \left(c_M^{1-\gamma} \left(\frac{A}{L} (nL)^{\alpha} \right)^{\gamma} \right)^{1-\sigma}$$

The terms on the diagonal (the principal minors of order one) are all less than or equal to zero which are the required signs. Since one of the diagonal elements is zero, only one minor of order two remains, namely

$$\Delta_2^3 = \frac{\alpha\gamma(1-(1-\sigma)(1-\gamma+\alpha\gamma))(1-\gamma)L\left(c_M^{1-\gamma}\left(\frac{A}{L}(nL)^{\alpha}\right)^{\gamma}\right)^{2(1-\sigma)}}{n^2 c_M^2} > 0.$$

The principal minor of order three (the Hessian's determinant) is zero. Therefore the conclusion is that *Mangasarian's* sufficiency conditions are met by the assumptions about parameter values.

A.2 Baseline Model: Parameter Restrictions for n^*

Valid values for n^* have to be inside the interval (0, 1). For $n > 0$ numerator and denominator must have the same sign. The steady-state value for n^* can be rearranged to yield:

$$n^* = \frac{\gamma(\rho-\lambda-(1-\sigma)((\alpha-1)\gamma\lambda+\gamma\nu+(1-\gamma)\frac{\mu}{\alpha})+\mu+\alpha\lambda)}{\rho-\lambda-(1-\sigma)((\alpha-1)\gamma\lambda+\gamma\nu+(1-\gamma)\frac{\mu}{\alpha})+\lambda(1-\gamma(1-\alpha))+(1-\gamma(1-\alpha))\frac{\mu}{\alpha}}.$$

It is obvious that by transversality condition (3.16) numerator as well as denominator are strictly positive.

For $n^* < 1$ we must have:

$$1 > \frac{\gamma(\rho-(1-\alpha)(1-(1-\sigma)\gamma)\lambda-(1-\sigma)(\gamma\nu+(1-\gamma)\frac{\mu}{\alpha})+\mu)}{\rho-(1-\alpha)\sigma\gamma\lambda-(1-\sigma)\gamma\nu+\sigma(1-\gamma)\frac{\mu}{\alpha}+\gamma\mu}$$

which simplifies to

$$(1-\gamma)(\rho-\lambda-(1-\sigma)((\alpha-1)\gamma\lambda+\gamma\nu+(1-\gamma)\frac{\mu}{\alpha})+\frac{\mu}{\alpha}+\lambda) > 0.$$

This inequality is always met by the transversality condition and the parameter restrictions. Hence, $n^* < 1$.

A.3 Baseline Model: Comparative Statics

Equation (3.17) can be written as $n^*(x) = f(x) / (f(x) + g(x))$, where from (3.17):

$$f(x) = \gamma(\rho-(1-\alpha)(1-(1-\sigma)\gamma)\lambda-(1-\sigma)(\gamma\nu+(1-\gamma)\frac{\mu}{\alpha})+\mu),$$

$$g(x) = (1-\gamma)(\rho+(1-\alpha)(1-\sigma)\gamma\lambda-(1-\sigma)(\gamma\nu+(1-\gamma)\frac{\mu}{\alpha})+\frac{\mu}{\alpha}).$$

Then the derivative of $n^*(x)$ with respect to x is:

$$\frac{\partial n^*}{\partial x} = \frac{f_x(f(x)+g(x)) - (g_x+f_x)f(x)}{(f(x)+g(x))^2}$$

where the denominator is positive. From rearranging the nominator one can see that

$$\text{(A.1)} \quad \frac{\partial n^*}{\partial x} \geq 0 \qquad \text{if} \qquad f_x g(x) - g_x f(x) \geq 0.$$

Asking now for the influence of the rate of technical progress in agriculture, ν, on n^*, we substitute the definitions for $f(x)$ and $g(x)$ into the last equation to obtain:

$$-\gamma^2(1-\sigma)(1-\gamma)(\rho+(1-\alpha)(1-\sigma)\gamma\lambda-(1-\sigma)(\gamma\nu+(1-\gamma)\frac{\mu}{\alpha})+\frac{\mu}{\alpha})$$

$$+\gamma^2(1-\sigma)(1-\gamma)(\rho-(1-\alpha)(1-(1-\sigma)\gamma)\lambda-(1-\sigma)(\gamma\nu+(1-\gamma)\frac{\mu}{\alpha})+\mu)$$

$$=-(1-\gamma)(1-\sigma)(1-\alpha)\gamma^2(\frac{\mu}{\alpha}+\lambda)$$

Thus, the sign of the derivative depends on σ. It is negative for $\sigma < 1$, zero for $\sigma = 1$, and positive for $\sigma > 1$.

By the same method the influence of the rate of technical progress in industry on the division of labor between the two sectors is obtained. Only, instead of taking the derivative with respect to ν, we take it now with respect to μ. Then from (A.1) we have for $f_x g(x) - g_x f(x)$:

$$\gamma(1-\gamma)(1-\frac{(1-\sigma)(1-\gamma)}{\alpha})(\rho+(1-\alpha)(1-\sigma)\gamma\lambda-(1-\sigma)(\gamma\nu+(1-\gamma)\frac{\mu}{\alpha})+\frac{\mu}{\alpha})$$

$$-\gamma(1-\gamma)(\frac{1-(1-\sigma)(1-\gamma)}{\alpha})(\rho-(1-\alpha)(1-(1-\sigma)\gamma)\lambda-(1-\sigma)(\gamma\nu+\frac{(1-\gamma)\mu}{\alpha})+\mu)$$

$$=\frac{(1-\alpha)(1-\gamma)\gamma}{\alpha}(\rho-\lambda-(1-\sigma)((\alpha-1)\gamma\lambda+\gamma\nu+\frac{(1-\gamma)\mu}{\alpha})+(1-\gamma)(1-\sigma)(\frac{\mu}{\alpha}+\lambda))$$

Comparing this result to the transversality condition (3.16), one can see that the above term is negative for $\sigma \leq 1$ but of ambiguous sign for $\sigma > 1$. For the special case that $\sigma > 1$ and $\lambda = 0$ the term is also negative.

Next, the influence of a change in the population growth rate λ can be analyzed by taking the derivative of equation (3.17) with respect to λ. We get for condition (A.1):

$$-\gamma(1-\alpha)(1-\gamma)(1-(1-\sigma)\gamma)(\rho+(1-\alpha)(1-\sigma)\gamma\lambda-(1-\sigma)(\gamma\nu+(1-\gamma)\frac{\mu}{\alpha})+\frac{\mu}{\alpha})$$

$$-\gamma^2(1-\alpha)(1-\gamma)(1-\sigma)(\rho-(1-\alpha)(1-(1-\sigma)\gamma)\lambda-(1-\sigma)(\gamma\nu+(1-\gamma)\frac{\mu}{\alpha})+\mu)$$

$$=-\gamma(1-\alpha)(1-\gamma)(\rho-\lambda-(1-\sigma)((\alpha-1)\gamma\lambda+\gamma\nu+(1-\alpha)\frac{\mu}{\alpha}))$$

$$+\frac{\mu}{\alpha}+(1-(1-\sigma)(1-\alpha)\gamma)\lambda$$

Again comparing this term to (3.16) the latter shows that it is always negative and therefore $\partial n^* / \partial \lambda < 0$.

Last, consider the derivative of n^* with respect to σ. The term $f_x g(x) - g_x f(x)$ is then given by:

$$\left((1-\alpha)\gamma\lambda - \left(\gamma\nu + (1-\gamma)\frac{\mu}{\alpha}\right)\right) \cdot$$

$$\left(\begin{array}{l} -\gamma(1-\gamma)\left(\rho + (1-\alpha)(1-\sigma)\gamma\lambda - (1-\sigma)\left(\gamma\nu + (1-\gamma)\frac{\mu}{\alpha}\right) + \frac{\mu}{\alpha}\right) \\ +\gamma(1-\gamma)\left(\rho - (1-\alpha)(1-(1-\sigma)\gamma)\lambda - (1-\sigma)\left(\gamma\nu + (1-\gamma)\frac{\mu}{\alpha}\right) + \mu\right) \end{array}\right)$$

$$= -\gamma(1-\alpha)(1-\gamma)\left(\lambda + \frac{\mu}{\alpha}\right)\left((1-\alpha)\gamma\lambda - \gamma\nu - (1-\gamma)\frac{\mu}{\alpha}\right)$$

The sign of this term depends on $(1-\alpha)\gamma\lambda - \gamma\nu - (1-\gamma)\frac{\mu}{\alpha}$. For $\lambda + \frac{\mu}{\alpha} = 0$ as well as for $\mu = 0$ and $\nu = (1-\alpha)\lambda$ it is zero.

A.4 Endogenous Growth: Transversality Conditions

The first transversality condition $\lim_{t \to \infty} e^{-\rho t} \theta_1 K = 0$ is met if

$$\rho > \frac{\dot{\theta}_1}{\theta_1} + \frac{\dot{K}}{K}.$$

Differentiating (4.5) and substituting for the growth rate of θ_1 leads to:

$$\rho > (1-\sigma)\gamma\eta\delta(1-u) + ((1-\sigma)(1-\gamma) - 1)\frac{\dot{c}_M}{c_M} + \frac{\dot{K}}{K}$$

Substituting u as well as the growth rates of c_M and K by their steady-state values given by equations (4.12) and (4.13) yields

$$\rho > (1-\sigma)\gamma\eta\delta + (1-\gamma)(1-\sigma)\frac{\mu}{\alpha}.$$

The second transversality condition $\lim_{t \to \infty} e^{-\rho t} \theta_2 A = 0$ is met if:

$$\rho > \frac{\dot{\theta}_2}{\theta_2} + \frac{\dot{A}}{A}$$

Substituting (4.9), (4.11), and (4.7) into the equation gives:

$$0 > -\frac{u\eta\delta}{\alpha}$$

Therefore u must be strictly positive which by (4.13) is the case in the steady-state if

$$\rho > (1-\sigma)\gamma\eta\delta + (1-\gamma)(1-\sigma)\frac{\mu}{\alpha}$$

since $1 - \alpha\gamma(1-\sigma) > 0$ by the assumptions about γ, α, and σ. Thus, both transversality conditions lead to the same parameter restriction.

A.5 Endogenous Growth: Parameter Restrictions from u^*

The steady-state value for u is given by (4.13) as

$$^* = \frac{\alpha(\rho - (1-\sigma)(\gamma\eta\delta + (1-\gamma)\frac{\mu}{\alpha})}{\eta\delta(1-\alpha\gamma(1-\sigma))}$$

It has already been shown in appendix A.4 that the first restriction, $u^* > 0$, is met by the transversality condition $\rho > (1-\sigma)\gamma\eta\delta + (1-\gamma)(1-\sigma)(\mu/\alpha)$.

The second restriction, $u^* < 1$, implies that:

$$\frac{\eta\delta(1-\alpha\gamma(1-\sigma))}{\alpha} > \rho - (1-\sigma)(\gamma\eta\delta + -(1-\gamma)\frac{\mu}{\alpha})$$

or, after rearranging:

$$\eta\delta > \alpha\rho - (1-\gamma)(1-\sigma)\mu$$

This condition can be interpreted as requiring a sufficiently large value for $\delta\eta$, the research efficiency times its elasticity in production. Taking both conditions together yields:

$$(1-\sigma)\gamma\eta\delta < \rho - (1-\gamma)(1-\sigma)\frac{\mu}{\alpha} < \frac{\eta\delta}{\alpha}$$

Both conditions together show that $\eta\delta$ must neither by too large nor too small.

A.6 Endogenous Growth: Steady-State Value for n

The steady-state value for n is obtained very much like in the basis model. First, substitution of (4.5) into (4.6) gives:

$$\frac{c_M}{K}\frac{(1-n)}{n}\frac{\gamma}{(1-\gamma)} = MK^{-\alpha}(1-n)^{\alpha}$$

Substitution of (4.12) into (4.8) leads to

$$\frac{c_M}{K} = MK^{-\alpha}(1-n)^{\alpha} - \frac{\dot{K}}{K} = MK^{-\alpha}(1-n)^{\alpha} - \frac{\mu}{\alpha}.$$

Combining these equations and solving for n yields an expression depending on $MK^{-\alpha}(1-n)^{\alpha}$:

$$n = \frac{\gamma(MK^{-\alpha}(1-n)^{\alpha} - \frac{\mu}{\alpha})}{MK^{-\alpha}(1-n)^{\alpha} - \gamma\frac{\mu}{\alpha}}$$

Now, differentiating equation (4.5) and combining the result with (4.10) leads to

$$\frac{\dot{\theta}_1}{\theta_1} = \rho - (1-\alpha)MK^{-\alpha}(1-n)^{\alpha} = (1-\sigma)\gamma\eta\delta(1-u) + ((1-\sigma)(1-\gamma) - 1)\frac{\mu}{\alpha}.$$

Substituting the steady-state fraction of agricultural labor u^* from equation (4.13) into this equation and rearranging yields

$$MK^{-\alpha}(1-n)^{\alpha} = \frac{\rho-(1-\sigma)\gamma\eta\delta-(1-\gamma)(1-\sigma)\frac{\mu}{\alpha}}{(1-\alpha)(1-\alpha\gamma(1-\sigma))}+\frac{\mu}{\alpha(1-\alpha)}$$

so that n^* is given by

$$n^* = \frac{\gamma(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma+\alpha^2\gamma)\frac{\mu}{\alpha}+\mu)}{\rho-(1-\sigma)\gamma\eta\delta+\sigma(1-\gamma+\alpha\gamma)\frac{\mu}{\alpha}+(1-\alpha)(1-\sigma)\gamma^2\mu}.$$

A.7 Endogenous Growth: Parameter Restrictions for n^*

Valid values for n^* have to lie inside the interval (0, 1). To ensure $n^* > 0$, numerator and denominator must have the same sign. The steady-state value for n^* can be rearranged to yield

$$n^* = \frac{\gamma(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma)\frac{\mu}{\alpha}+(1-\alpha\gamma(1-\sigma))\mu)}{\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma)\frac{\mu}{\alpha}+(1-\gamma+\alpha\gamma)(1-(1-\sigma)\alpha\gamma)\frac{\mu}{\alpha}}.$$

It is obvious that by the transversality condition numerator as well as denominator are positive.

To satisfy the second constraint, $n^* < 1$, we must have:

$$1 > \frac{\gamma(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma+\alpha^2\gamma)\frac{\mu}{\alpha}+\mu)}{\rho-(1-\sigma)\gamma\eta\delta+\sigma(1-\gamma+\alpha\gamma)\frac{\mu}{\alpha}+(1-\alpha)(1-\sigma)\gamma^2\mu}$$

which simplifies to

$$(1-\gamma)(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma+\alpha^2\gamma)\frac{\mu}{\alpha}+\mu+(1-(1-\sigma)\alpha\gamma)\frac{\mu}{\alpha}) > 0.$$

This inequality is met by the transversality condition as well as the parameter restrictions and hence $n^* < 1$.

A.8 Endogenous Growth: Differential equations for n, u, and z_2

First, the differential equation for z_2 can be obtained by differentiating the definition for z_2, replacing the growth rate of K by equation (4.8), and eliminating z_1:

$$= z_2(\mu-\alpha z_2(1-n)^{\alpha-1}(\frac{\gamma-n}{\gamma}));$$

Now combining equations (4.5) and (4.7), differentiating the result and replacing the growth rates of the shadow prices by equations (4.10) and (4.11) leads after some rearrangements to:

$$\frac{\dot{c}_M}{c_M} = \frac{\dot{u}}{u} - \frac{u\eta\delta}{\alpha} + (1-\alpha)\, z_2\, (1-n)^{\alpha}$$

Differentiating equation (4.5), solving for the growth rate of c_M and equating the result to the previous equation yields a differetial equation for u which, however, still depends on the growth rate of n:

$$\frac{\dot{u}}{u} = \frac{\rho - (1-\alpha)\,(1-\sigma)\,(1-\gamma)\, z_2\,(1-n)^{\alpha} - \gamma(1-\sigma)\,\eta\delta\,(1-u)}{(1-\sigma)\,\gamma\,(\alpha-1) - \sigma}$$
$$+ \frac{\frac{u\eta\delta}{\alpha}\,(\gamma\,(\sigma-1) - \sigma) - \gamma\,(1-\sigma)\,\alpha\frac{\dot{n}}{n}}{(1-\sigma)\,\gamma\,(\alpha-1) - \sigma}$$

Combination of equations (4.5) and (4.6), differentiation, and some substitutions lead to a similar equation for the growth rate of n:

$$\frac{\dot{n}}{n} = \frac{(1-n)\frac{\dot{u}}{u} + \frac{(1-\alpha)\,(1-\gamma)}{\gamma} z_2 n\,(1-n)^{\alpha} - \frac{u\eta\delta}{\alpha}\,(1-n) - \mu\,(1-n)}{1-\alpha n}$$

The last two differential equations can be combined and solved for the growth rates of n and u:

$$\frac{\dot{u}}{u} = \frac{(1-\alpha n)\,(\alpha\rho - (1-\sigma)\,\alpha\eta\delta\gamma - \delta\eta\sigma u + \alpha\delta\eta\gamma u\,(1-\sigma))}{\alpha((\alpha-1)\,\gamma\,(1-\sigma) + \alpha\gamma\,(1-\sigma)\,(1-\alpha n) - (1-\alpha n))}$$
$$+ \frac{(1-\sigma)\,(\alpha^2\gamma\mu\,(1-n) - \delta\eta\gamma u\,(1-\alpha) - \alpha\,(1-\alpha)\,(1-\gamma)\, z_2\,(1-n)^{\alpha})}{\alpha((\alpha-1)\,\gamma\,(1-\sigma) + \alpha\gamma\,(1-\sigma)\,(1-\alpha n) - (1-\alpha n))}$$

$$\frac{\dot{n}}{n} = \frac{(1-n)\,(\alpha\rho - (1-\sigma)\,\alpha\eta\delta\gamma - \delta\eta\sigma u + \alpha\delta\eta\gamma u\,(1-\sigma))}{\alpha((\alpha-1)\,\gamma\,(1-\sigma) + \alpha\gamma\,(1-\sigma)\,(1-\alpha n) - (1-\alpha n))} +$$
$$+ \frac{(1-\alpha)\,(1-\gamma)\, z_2 n\,(1-n)^{\alpha}}{\gamma\,(1-\alpha n)} - \frac{\frac{u\eta\delta}{\alpha}\,(1-n) + \mu\,(1-n)}{(1-\alpha n)}$$
$$+ \frac{(1-\sigma)\,(\alpha^2\gamma\mu\,(1-n) - \delta\eta\gamma u\,(1-\alpha) - \alpha\,(1-\alpha)\,(1-\gamma)\, z_2\,(1-n)^{\alpha})}{\alpha((\alpha-1)\,\gamma\,(1-\sigma) + \alpha\gamma\,(1-\sigma)\,(1-\alpha n) - (1-\alpha n))}$$

For the special case that $\sigma = 1$ these two differential equations simplify to:

$$\frac{\dot{u}}{u} = -\frac{\alpha\rho - \eta\delta u}{\alpha}$$

$$\dot{n} = \frac{n\,(1-n)}{(1-\alpha n)}\left[-\rho - \mu + (1-\alpha)\, n z_2\,(1-n)^{\alpha-1}\frac{(1-\gamma)}{\gamma}\right]$$

A.9 Endogenous Growth: Comparative Statics with Externality Wedge

Similarly to the derivatives calculated in A.3 the sign is positive if $f_x g(x) - g_x f(x) \geq 0$ with the only difference that n^{**} is now given by (4.23) instead of (3.17). Thus, these terms are now:

$$f(x) = \gamma\left(\rho - \gamma(1-\sigma)\eta\delta - (1-\sigma)\left(1-\gamma+\alpha^2\gamma\frac{\eta}{\eta_2}\right)\frac{\mu}{\alpha} + \mu\right)$$

$$g(x) = (1-\gamma)\left(\rho - \gamma(1-\sigma)\eta\delta - (1-\sigma)\left(1-\gamma+\alpha\gamma\frac{\eta}{\eta_2}\right)\frac{\mu}{\alpha} + \frac{\mu}{\alpha}\right)$$

Then $f_x g(x) - g_x f(x)$ for the derivative with respect to the externality wedge is (cf. appendix A.3)

$$-\gamma^2(1-\sigma)\alpha(1-\gamma)\mu\left(\rho - \gamma(1-\sigma)\eta\delta - (1-\sigma)\left(1-\gamma+\alpha\gamma\frac{\eta}{\eta_2}\right)\frac{\mu}{\alpha} + \frac{\mu}{\alpha}\right)$$

$$+\gamma^2(1-\sigma)(1-\gamma)\mu\left(\rho - \gamma(1-\sigma)\eta\delta - (1-\sigma)\left(1-\gamma+\alpha^2\gamma\frac{\eta}{\eta_2}\right)\frac{\mu}{\alpha} + \mu\right).$$

Simplification leads to

$$(1-\alpha)\gamma^2(1-\sigma)(1-\gamma)\mu\left(\rho - \gamma(1-\sigma)\eta\delta - (1-\sigma)(1-\gamma)\frac{\mu}{\alpha}\right).$$

Since the last term in brackets is positive by the transversality condition, the sign depends on σ. The derivative is positive for $\sigma < 1$, zero for $\sigma = 1$, and negative for $\sigma > 1$.

A.10 Endogenous Growth: Comparative Statics for u^*

The derivative of u^* given by (4.13) with respect to μ can be calculated as

$$\frac{\partial u^*}{\partial \mu} = -\frac{(1-\gamma)(1-\sigma)}{\eta\delta(1-\alpha\gamma(1-\sigma))}.$$

By the assumptions about α and γ this term is negative for $\sigma < 1$, zero for $\sigma = 1$, and positive for $\sigma > 1$.

Differentiating u^* with respect to δ yields:

$$\frac{\partial u^*}{\partial \delta} = -\frac{\alpha\gamma\eta^2\delta(1-\sigma) + \eta\alpha\left(\rho - \gamma(1-\sigma)\eta\delta - (1-\gamma)(1-\sigma)\frac{\mu}{\alpha}\right)}{\eta^2\delta^2(1-\alpha\gamma(1-\sigma))}$$

$$= -\frac{\alpha\left(\rho - (1-\gamma)(1-\sigma)\frac{\mu}{\alpha}\right)}{\eta\delta^2(1-\alpha\gamma(1-\sigma))} < 0$$

because of the transversality condition. It is easy to see that also $\partial u / \partial \eta < 0$.

The derivative of u^* with respect to σ has the same sign as:

$$\frac{\alpha\gamma((\eta\delta-\alpha\rho+\alpha(1-\gamma)(1-\sigma)\frac{\mu}{\alpha})+(1-\gamma)\mu+\alpha\gamma(1-\sigma)\eta\delta)}{\eta\delta(1-\alpha\gamma(1-\sigma))}$$

From the assumptions about the parameter values the denominator is positive. From condition (4.16) the term in brackets in the numerator is positive, too. Thus, for $\sigma < 1$ as well as for $\sigma = 1$ the derivative has a positive sign. For $\sigma > 1$ it is ambiguous.

A.11 Endogenous Growth: Comparative Statics for n^*

Equation (4.17) can be written as $n(x) = f(x) / (f(x) + g(x))$, where equation (4.17) leads after some rearrangements to:

$$f(x) = \gamma(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma+\alpha^2\gamma)\frac{\mu}{\alpha}+\mu)$$

$$g(x) = (1-\gamma)(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma+\alpha\gamma)\frac{\mu}{\alpha}+\frac{\mu}{\alpha})$$

Turning first to μ, we get for $f_x g(x) - g_x f(x)$ (cf. appendix A.3):

$$(1-\gamma)\gamma\left(1-\frac{(1-\sigma)(1-\gamma+\alpha^2\gamma)}{\alpha}\right)(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma+\alpha\gamma)\frac{\mu}{\alpha}+\frac{\mu}{\alpha})$$
$$-(1-\gamma)\gamma(\frac{1}{\alpha}-\frac{(1-\sigma)(1-\gamma+\alpha\gamma)}{\alpha})(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma+\alpha^2\gamma)\frac{\mu}{\alpha}+\mu)$$

Simplification leads to

$$-\frac{(1-\alpha)(1-\alpha\gamma(1-\sigma))\gamma(1-\gamma)}{\alpha}(\rho-(1-\sigma)\gamma\eta\delta).$$

According to the transversality condition the term in brackets is positive and therefore $\partial n^*/\partial\mu < 0$.

Next consider a change in δ. The term for $f_x g(x) - g_x f(x)$ is:

$$-\gamma^2(1-\sigma)\eta(1-\gamma)(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma+\alpha\gamma)\frac{\mu}{\alpha}+\frac{\mu}{\alpha})$$

$$+(1-\gamma)(1-\sigma)\gamma^2\eta(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)(1-\gamma+\alpha^2\gamma)\frac{\mu}{\alpha}+\mu)$$

$$=-(1-\alpha)\gamma^2(1-\sigma)\eta(1-\gamma)\frac{\mu}{\alpha}(1-\alpha\gamma(1-\sigma))$$

The sign of this term depends on σ. The derivative is: negative for $\sigma < 1$, zero for $\sigma = 1$, and positive for $\sigma > 1$. It is again trivial to show that also $\partial n^*/\partial\eta < 0$.

The sign of the derivative of n^* with respect to σ is equal to the sign of:

$$\gamma(\gamma\eta\delta + (1-\gamma+\alpha^2\gamma)\frac{\mu}{\alpha})\,(1-\gamma)\,(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)\,(1-\gamma+\alpha\gamma)\frac{\mu}{\alpha}+\frac{\mu}{\alpha})$$

$$-(1-\gamma)\,(\gamma\eta\delta + (1-\gamma+\alpha\gamma)\frac{\mu}{\alpha})\gamma(\rho-(1-\sigma)\gamma\eta\delta-(1-\sigma)\,(1-\gamma+\alpha^2\gamma)\frac{\mu}{\alpha}+\mu)$$

$$= -(1-\alpha)\gamma(1-\gamma)\,(\alpha\gamma\rho-\gamma\eta\delta-\gamma\alpha(1-\sigma)\,(1-\gamma)\frac{\mu}{\alpha}-(1-\gamma)\,(1-\alpha\gamma(1-\sigma))\frac{\mu}{\alpha})$$

The sum of the first three terms in the third bracket is negative by condition (4.16). The last term, which is subtracted, is positive. Therefore $\partial n^* / \partial\sigma > 0$, except when $\mu = 0$ where the derivative is zero.

A.12 Technology Adoption: Transversality Conditions

The first transversality condition $\lim_{t\to\infty} e^{-\rho t}\theta_1 K = 0$ is met if

$$\rho > \frac{\dot{\theta}_1}{\theta_1} + \frac{\dot{K}}{K}.$$

Differentiating equation (5.10) and substituting for the growth rate of θ_1 leads to:

$$\rho > (1-\sigma)\gamma\eta\frac{\dot{A}_A}{A_A} + ((1-\sigma)\,(1-\gamma)-1)\frac{\dot{c}_M}{c_M} + \frac{\dot{K}}{K}$$

Replacing the growth rates of A_A, c_M and K by their steady-state values given by (5.17) and (5.18) yields:

$$\rho > (1-\sigma)\,(\gamma\eta\nu + (1-\gamma)\frac{\mu}{\alpha})$$

The second transversality condition $\lim_{t\to\infty} e^{-\rho t}\theta_2 A_A = 0$ is met if

$$\rho > \frac{\dot{\theta}_2}{\theta_2} + \frac{\dot{A}_A}{A_A}.$$

Differentiating equation (5.12) and substituting the result into the equation together with the steady-state growth rates of c_M and A_A yields

$$\rho > (1-\sigma)\,(\gamma\eta\nu + (1-\gamma)\frac{\mu}{\alpha}).$$

Thus, both transversality conditions lead to the same parameter restriction.

A.13 Technology Adoption: Steady-State Value for *n*

The steady-state value for n is obtained very much like in the basic model. First, substitution of equation (5.10) into (5.11) gives:

$$\frac{c_M}{K}\frac{(1-n)}{n}\frac{\gamma}{(1-\gamma)} = MK^{-\alpha}(1-n)^{\alpha}$$

Substitution of equation (5.17) into (5.13) leads after rearranging to

$$\frac{c_M}{K} = MK^{-\alpha}(1-n)^{\alpha} - \frac{\dot{K}}{K} = MK^{-\alpha}(1-n)^{\alpha} - \frac{\mu}{\alpha}.$$

Combining these equations and solving for n yields, depending on $MK^{-\alpha}(1-n)^{\alpha}$:

$$n = \frac{\gamma\left(MK^{-\alpha}(1-n)^{\alpha} - \frac{\mu}{\alpha}\right)}{MK^{-\alpha}(1-n)^{\alpha} - \gamma\frac{\mu}{\alpha}}$$

Now differentiating equation (5.10) and combining the result with (5.15) leads to:

$$\frac{\dot{\theta}_1}{\theta_1} = \rho - (1-\alpha)MK^{-\alpha}(1-n)^{\alpha} = (1-\sigma)\gamma\eta\nu + ((1-\sigma)(1-\gamma)-1)\frac{\mu}{\alpha}$$

Rearranging yields

$$MK^{-\alpha}(1-n)^{\alpha} = \frac{\rho-(1-\sigma)\gamma\eta\nu - ((1-\sigma)(1-\gamma)-1)\frac{\mu}{\alpha}}{(1-\alpha)}$$

so that n^* is given by

$$n^* = \frac{\gamma\left(\rho - (1-\sigma)\left(\gamma\eta\nu + (1-\gamma)\frac{\mu}{\alpha}\right) + \mu\right)}{\rho - (1-\sigma)\gamma\eta\nu + \sigma(1-\gamma)\frac{\mu}{\alpha} + \gamma\mu}.$$

A.14 Technology Adoption Conditions for Feasible u^*

The first condition to ensure $u^* > 0$ is:

$$1 + \frac{B + \frac{\eta\nu}{\alpha}}{2\delta} > \sqrt{\frac{\left(B + \frac{\eta\nu}{\alpha} + 2\delta\right)^2}{4\delta^2} - \frac{B}{\delta} - 1}$$

which can be transformed into:

$$\frac{B + \frac{\eta\nu}{\alpha} + 2\delta}{2\delta} > \sqrt{\frac{\left(B + \frac{\eta\nu}{\alpha} + 2\delta\right)^2}{4\delta^2} - \frac{B}{\delta} - 1}.$$

Obviously the first term under the square root equals the term on the left-hand side squared. Since $B > 0$ by the transversality condition, the term under the root is necessarily smaller than the square of the right-hand side term. Thus, the condition is always met.

The second condition requires that the term under the square root must be positive.

$$\frac{\left(B+\frac{\eta\nu}{\alpha}+2\delta\right)^2}{4\delta^2}-\frac{B}{\delta}-1>0$$

This can be simplified to yield:

$$B^2+2B\frac{\eta\nu}{\alpha}+4\frac{\eta\nu}{\alpha}\delta+\frac{\eta^2\nu^2}{\alpha^2}>0$$

which, since $B > 0$ by the transversality condition, is always true.

A.15 Technology Adoption: Comparative Statics

The derivative of u^* with respect to δ can simply be calculated from equation (5.21) which defines u^* as:

$$u^* = 1+\frac{B+\frac{\eta\nu}{\alpha}}{2\delta}-\sqrt{\frac{\left(B+\frac{\eta\nu}{\alpha}+2\delta\right)^2}{4\delta^2}-\frac{B}{\delta}-1}$$

where $B = \rho+\nu-(1-\sigma)\,(\gamma\eta\nu+(1-\gamma)\,\frac{\mu}{\alpha})$.

We get:

$$\frac{\partial u^*}{\partial \delta} = -\frac{B+\frac{\eta\nu}{\alpha}}{2\delta^2}+\frac{\alpha^2B^2+2\alpha B\eta\nu+2\alpha\delta\eta\nu+\eta^2\nu^2}{4\alpha^2\delta^3\sqrt{\frac{\left(B+\frac{\eta\nu}{\alpha}+2\delta\right)^2}{4\delta^2}-\frac{B}{\delta}-1}}$$

This term is positive, if

$$\frac{\alpha^2B^2+2\alpha B\eta\nu+2\alpha\delta\eta\nu+\eta^2\nu^2}{2\alpha^2\delta\left(B+\frac{\eta\nu}{\alpha}\right)}>\sqrt{\frac{\left(B+\frac{\eta\nu}{\alpha}+2\delta\right)^2}{4\delta^2}-\frac{B}{\delta}-1}.$$

This condition can be simplified to

$$\frac{\eta^2\nu^2}{\alpha^2\left(B+\frac{\eta\nu}{\alpha}\right)^2}>0$$

which is met since $B > 0$ by the transversality condition (5.19). Therefore $\frac{\partial u^*}{\partial \delta}>0$.

The derivative of a^* can be obtained similarly. From equation (5.22) we have

$$a^* = \frac{\delta(1-u^*)}{\nu+\delta(1-u^*)}.$$

The derivative of $\delta(1-u^*)$ is:

$$\frac{\partial\delta(1-u^*)}{\partial\delta} = \frac{\eta\nu}{\sqrt{\alpha^2B^2+2\alpha B\eta\nu+4\alpha\delta\eta\nu+\eta^2\nu^2}}$$

which is positive since $B > 0$. Since an increase in δ raises the nominator of the a^*-equation more than its denominator, this implies $\partial a^* / \partial\delta > 0$ as long as $\nu > 0$.

A.16 Technology Adoption: Differential Equations for Endogenous Model

First, the differential equation for a is already given as:

$$\text{(A.2)} \quad \frac{\dot{a}}{a} = \delta(1-u)\frac{(1-a)}{a} - \nu$$

Next, we define again two new variables, $z_1 = c_M / K$ as control-like and $z_2 = M / K^{\alpha}$ as state-like variable. From equations (5.10) and (5.11) z_1 is always given as:

$$\text{(A.3)} \quad z_1 = \frac{(1-\gamma)}{\gamma}z_2 n(1-n)^{\alpha-1}$$

Differentiating (5.12) and making use of equations (5.14), (5.16), and (A.2) leads to

$$\frac{\dot{u}}{u} = \frac{\eta\delta u}{\alpha}\frac{(1-a)}{a} - \frac{\nu}{(1-a)} - \rho.$$

Differentating equation (5.10) yields:

$$\frac{\dot{c}_M}{c_M} = (1-\alpha)z_2(1-n)^{\alpha} - \rho$$

Finally, differentiating equation (A.3), using (5.13) as well as the previous equation yields

$$\frac{\dot{n}}{n} = \frac{(1-n)}{(1-\alpha n)}\left[-\rho-\mu+(1-\alpha)z_2(1-n)^{\alpha-1}\left(\frac{1-\gamma}{\gamma}\right)\right].$$

A.17 Engel's Law: Income Elasticities

The income elasticities for the Stone-Geary utility function are derived from a simple maximization problem. Consider $u(c_A, c_M) = a\ln(c_A-\xi) + b\ln(c_M)$, a more general form. This function has to be maximized subject to an income constraint. The Lagrangian is therefore:

$$L = a\ln(c_A-\xi) + b\ln(c_M) + \lambda(y-c_A-pc_M)$$

with y denoting income and p the price of widgets in terms of food. This Lagrangian yields the solution equations:

$$\frac{\partial L}{\partial c_A} = \frac{a}{c_A - \xi} - \lambda = 0$$

$$\frac{\partial L}{\partial c_M} = \frac{b}{c_M} - \lambda p = 0$$

$$\frac{\partial L}{\partial \lambda} = y - c_A - pc_M = 0$$

Combining these equations to eliminate λ leads to the demand functions $c_A = (ay + b\xi)/(a+b)$, $c_M = b(y-\xi)/(p(a+b))$ and thus the income elasticities:

$$\varepsilon_A = \frac{ay}{ay + b\xi}, \qquad \varepsilon_B = \frac{y}{y - \xi}.$$

A.18 Engel's Law: Modified Differential Equations

First, substitute equation (6.4) into equation (6.5) to eliminate θ and use the variable definitions to get

$$\text{(A.4)} \quad z_1 = \frac{(1-\gamma)}{\gamma} z_2 n (1-n)^{\alpha - 1} \frac{An^{\alpha} - \xi}{An^{\alpha}}.$$

Thus, z_1 is given at every moment in time as a combination of the remaining variables. Differentiating the definitions for z_2 while making use of the previous equation and (6.6) leads to:

$$\text{(A.5)} \quad \dot{z}_2 = z_2 \left(\mu - \alpha z_2 (1-n)^{\alpha - 1} \left(\frac{An^{\alpha}(\gamma - n) + n\xi(1-\gamma)}{An^{\alpha}\gamma} \right) \right)$$

Next, differerentiating (A.4) while using (A.5) as well as the definition for z_1 leads to:

$$\frac{\dot{c}_M}{c_M} - \frac{\dot{K}}{K} = \frac{\dot{z}_2}{z_2} + \frac{\dot{n}}{n} - \frac{(\alpha - 1)\dot{n}}{(1-n)} + \alpha \frac{\dot{n}}{n} \frac{An^{\alpha}}{An^{\alpha} - \xi} - \alpha \frac{\dot{n}}{n} + \nu \frac{An^{\alpha}}{An^{\alpha} - \xi} - \nu$$

Substituting the growth rates of k and c_M by using equations (6.4), (6.6), and (6.7) and the growth rate of z_2 from equation (A.5) yields after some rearrangements

$$\dot{n} = \frac{n(1-n)\left[(1-\alpha) z_2 (1-n)^{\alpha - 1} \left(\frac{n(1-\gamma)(An^{\alpha} - \xi)}{An^{\alpha}\gamma} \right) - \rho - \mu - \frac{\nu\xi}{An^{\alpha} - \xi} \right]}{(1 - \alpha n) + \alpha(1-n)\frac{\xi}{An^{\alpha} - \xi}}.$$

A.19 Nutrition-Productivity Relationship: Differential Equations

The first model modification, a static relationship in agriculture, is derived just like the second one in the main text and yields the following equations:

$$\Pi = \pi\left(1 - 1/\left(An^{\alpha}\right)\right)$$

$$z_1 = \frac{(1-\gamma)}{\gamma} z_2 n (1-n)^{\alpha-1}$$

$$\dot{z}_2 = z_2\left(\mu - \alpha z_2 (1-n)^{\alpha-1}\frac{(\gamma-n)}{\gamma}\right)$$

$$\dot{n} = \frac{n(1-n)\left[(1-\sigma)\left(\mu(1-\gamma) + \frac{\gamma\nu\pi}{\Pi}\right) - \mu - \rho\right]}{\sigma(1-\alpha n) + (1-\sigma)\gamma(1-\alpha) - (1-\sigma)\gamma\alpha\left(\frac{\pi}{\Pi} - 1\right)(1-n)}$$

$$+\frac{(1-\alpha)(1-\gamma) n z_2 (1-n)^{\alpha-1}\frac{(\sigma n + \gamma(1-\sigma))}{\gamma(1-n)}}{\sigma(1-\alpha n) + (1-\sigma)\gamma(1-\alpha) - (1-\sigma)\gamma\alpha\left(\frac{\pi}{\Pi} - 1\right)(1-n)}$$

For the next modification, a dynamic relationship in industry, the state-like variable z_2 is defined as $z_2 = M / K^{\alpha(1-\Pi)}$. This leads to the equations:

$$\Pi = \left(\frac{\pi A n^{\alpha}}{\pi + A n^{\alpha}}\right)$$

$$z_1 = \frac{(1-\gamma)}{\gamma} z_2 n (1-n)^{\alpha-1}$$

$$\dot{z}_2 = z_2\left(\mu - \alpha(1-\Pi) z_2 (1-n)^{\alpha-1}\frac{(\gamma-n)}{\gamma}\right)$$

$$\dot{n} = \frac{n(1-n)\left[(1-\sigma)(\mu(1-\gamma) + \gamma\nu) - \mu - \rho + z_2(1-\alpha)(1-\gamma)(1-\sigma)(1-n)^{\alpha-1}\right]}{(\sigma(1-\alpha n) + (1-\sigma)\gamma(1-\alpha))}$$

$$+\frac{z_2 n (1-n)^{\alpha}\left(n\sigma(1-\alpha-\gamma+\alpha\gamma) + \alpha\Pi(n-\gamma)(\gamma(1-\sigma)+\sigma)\right)}{\gamma(\sigma(1-\alpha n) + (1-\sigma)\gamma(1-\alpha))}$$